

Mathletics

Series



Student



$6 \times 7 = 42$ $6 \times 7 = 42$

$6 \times 7 = 42$ $6 \times 7 = 42$

$6 \times 7 = 42$ $6 \times 7 = 42$

Multiplication and Division

My name _____



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Series Author:

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Multiplication facts – 5 and 10 times tables

The 5 and 10 times tables are easier if you learn them together.

1 Answer the 5 times table:

$1 \times 5 = \square$

$2 \times 5 = \square$

$3 \times 5 = \square$

$4 \times 5 = \square$

$5 \times 5 = \square$

$6 \times 5 = \square$

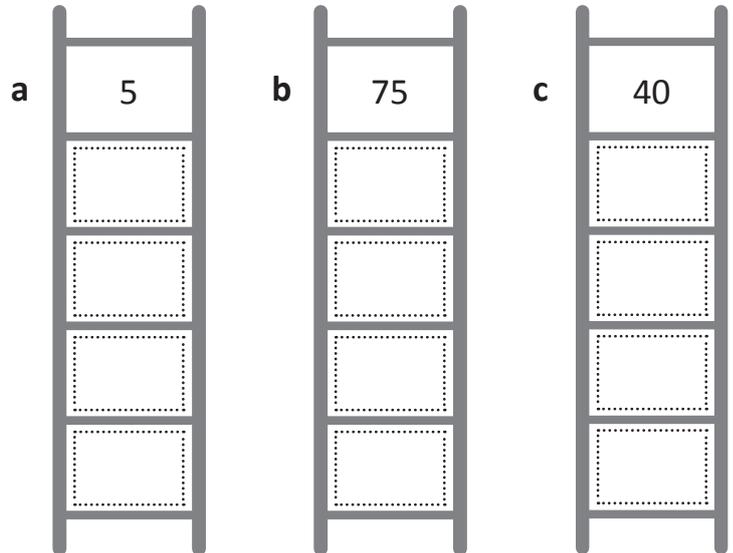
$7 \times 5 = \square$

$8 \times 5 = \square$

$9 \times 5 = \square$

$10 \times 5 = \square$

2 Count in 5s down the ladders:



3 Fill in the missing number for each times table fact:

$\square \times 5 = 25$

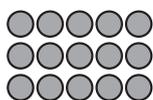
$\square \times 5 = 45$

$\square \times 5 = 30$

$\square \times 5 = 50$

$\square \times 5 = 35$

$\square \times 5 = 40$



$3 \times 5 = 15$



$5 \times 3 = 15$

Turnaround facts are the times tables turned around!



REMEMBER

4 Complete the 5 times table turnarounds.

$5 \times 8 = \square$

$5 \times 3 = \square$

$5 \times 10 = \square$

$5 \times 4 = \square$

Multiplication facts – 2 and 4 times tables

The 2 and 4 times tables are good facts to learn together.

1 Complete the skip counting pattern of 2:



2 Answer the 2 times table. One is in order, the other is mixed up.

$1 \times 2 = \square$

$2 \times 2 = \square$

$3 \times 2 = \square$

$4 \times 2 = \square$

$5 \times 2 = \square$

$6 \times 2 = \square$

$7 \times 2 = \square$

$8 \times 2 = \square$

$9 \times 2 = \square$

$10 \times 2 = \square$

$7 \times 2 = \square$

$10 \times 2 = \square$

$6 \times 2 = \square$

$8 \times 2 = \square$

$1 \times 2 = \square$

$9 \times 2 = \square$

$4 \times 2 = \square$

$3 \times 2 = \square$

$2 \times 2 = \square$

$5 \times 2 = \square$

3 It is useful to be able to multiply numbers above 10 by 2. Try these:

$11 \times 2 = \square$

$12 \times 2 = \square$

$13 \times 2 = \square$

$14 \times 2 = \square$

$15 \times 2 = \square$

$16 \times 2 = \square$

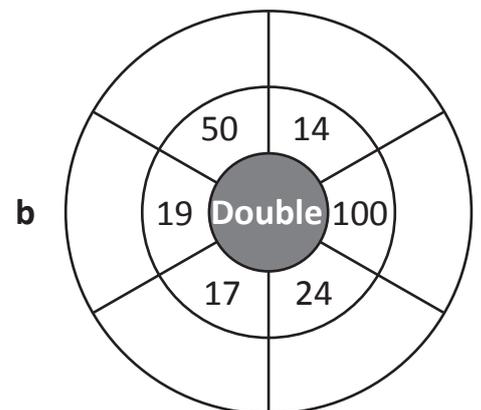
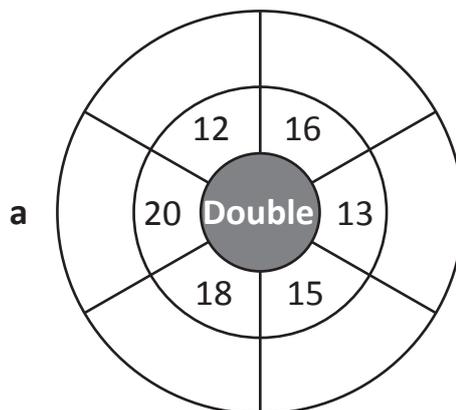
$17 \times 2 = \square$

$18 \times 2 = \square$

$19 \times 2 = \square$

$20 \times 2 = \square$

4 Complete these doubling wheels as quickly as you can. Multiplying by 2 is the same as doubling.



Multiplication facts – 2 and 4 times tables

Now for the 4 times table. The 4 times table is just double the 2 times table. This is handy to remember if you forget a 4 times table fact.

5 The 2 times table should be easier, so complete it first. Then double each of the 2 times table facts to get the 4 times table facts:

$1 \times 2 =$	<input type="text"/>	$1 \times 4 =$	<input type="text"/>
$2 \times 2 =$	<input type="text"/>	$2 \times 4 =$	<input type="text"/>
$3 \times 2 =$	<input type="text"/>	$3 \times 4 =$	<input type="text"/>
$4 \times 2 =$	<input type="text"/>	$4 \times 4 =$	<input type="text"/>
$5 \times 2 =$	<input type="text"/>	$5 \times 4 =$	<input type="text"/>
$6 \times 2 =$	<input type="text"/>	$6 \times 4 =$	<input type="text"/>
$7 \times 2 =$	<input type="text"/>	$7 \times 4 =$	<input type="text"/>
$8 \times 2 =$	<input type="text"/>	$8 \times 4 =$	<input type="text"/>
$9 \times 2 =$	<input type="text"/>	$9 \times 4 =$	<input type="text"/>
$10 \times 2 =$	<input type="text"/>	$10 \times 4 =$	<input type="text"/>

6 Write the missing numbers for these 4 times table facts:

a $\times 4 = 8$

b $\times 4 = 16$

c $\times 4 = 40$

d $\times 4 = 24$

e $\times 4 = 12$

f $\times 4 = 36$

g $\times 4 = 20$

h $\times 4 = 28$

7 Use the hint to get the answer. Then fill in the missing digit to make the 4 times table fact complete:

a **Hint: Double 16**

$\times 4 =$

b **Hint: Double 12**

$\times 4 =$

c **Hint: Double 18**

$\times 4 =$

8 Look at the numbers in the grid and circle 3 numbers that would make a multiplication fact. Look for $\times 2$ and $\times 4$ facts. They are either left to right or top to bottom. The first one has been done for you. There are 10 to find.

4	3	12	4	8	32
4	1	3	2	7	1
16	5	3	8	2	9
3	4	6	24	14	4
2	8	16	7	9	36
9	2	18	10	2	20

Multiplication facts – 8 times table

Here is the 8 times table. You can double the 4 times table to get the 8 times table.

- 1 Complete the 4 times table as quickly as you can. Then after you have checked them, double them to complete the 8 times table facts:

$1 \times 4 = \square$

$2 \times 4 = \square$

$3 \times 4 = \square$

$4 \times 4 = \square$

$5 \times 4 = \square$

$6 \times 4 = \square$

$7 \times 4 = \square$

$8 \times 4 = \square$

$9 \times 4 = \square$

$10 \times 4 = \square$

$1 \times 8 = \square$

$2 \times 8 = \square$

$3 \times 8 = \square$

$4 \times 8 = \square$

$5 \times 8 = \square$

$6 \times 8 = \square$

$7 \times 8 = \square$

$8 \times 8 = \square$

$9 \times 8 = \square$

$10 \times 8 = \square$

- 2 Use double, double and double again for these problems:

a $6 \times 8 = \square$

b $4 \times 8 = \square$

c $9 \times 8 = \square$

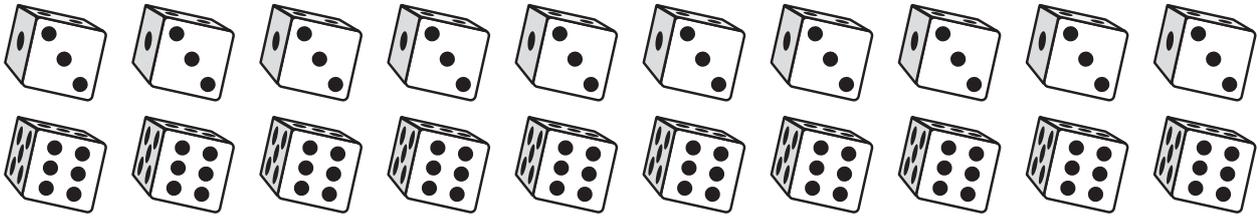


If you get stuck on the 8s, think double, double and double again.
For example, 3×8
Think: double 3 is 6
double 6 is 12
double 12 is 24

- 3 On Mia's calculator, the 8 key is broken. Show her the steps she could follow to find the answer to 16×8 . Use a calculator to test the steps.

Multiplication facts – 3 and 6 times tables

Here are the 3 times and 6 times tables together. Can you think of why it's better to learn these facts together?



1 Use the picture of the dice above to complete both the 3 times table and the 6 times table:

$1 \times 3 =$	<input type="text"/>	$1 \times 6 =$	<input type="text"/>
$2 \times 3 =$	<input type="text"/>	$2 \times 6 =$	<input type="text"/>
$3 \times 3 =$	<input type="text"/>	$3 \times 6 =$	<input type="text"/>
$4 \times 3 =$	<input type="text"/>	$4 \times 6 =$	<input type="text"/>
$5 \times 3 =$	<input type="text"/>	$5 \times 6 =$	<input type="text"/>
$6 \times 3 =$	<input type="text"/>	$6 \times 6 =$	<input type="text"/>
$7 \times 3 =$	<input type="text"/>	$7 \times 6 =$	<input type="text"/>
$8 \times 3 =$	<input type="text"/>	$8 \times 6 =$	<input type="text"/>
$9 \times 3 =$	<input type="text"/>	$9 \times 6 =$	<input type="text"/>
$10 \times 3 =$	<input type="text"/>	$10 \times 6 =$	<input type="text"/>

2 Now try these mixed up:

- a $3 \times 6 =$
- b $4 \times 3 =$
- c $8 \times 3 =$
- d $9 \times 6 =$
- e $4 \times 6 =$
- f $5 \times 3 =$
- g $8 \times 6 =$
- h $9 \times 3 =$
- i $5 \times 6 =$

3 Fill in the missing digits to make these times table facts complete:

- a $3 \times 3 =$
- b $\times 2 = 6$
- c $\times 3 = 18$
- d $6 \times$ $= 36$
- e $3 \times$ $= 24$
- f $\times 6 = 60$
- g $\times 9 = 27$
- h $6 \times$ $= 42$
- i $9 \times$ $= 54$
- j $5 \times$ $= 30$
- k $\times 6 = 48$
- l $7 \times$ $= 21$

Multiplication facts – 3 and 6 times tables

4 Match the answers to the questions. Each answer has two matching questions.

4×6

16×3

3×8

3×10

8×6



3×4

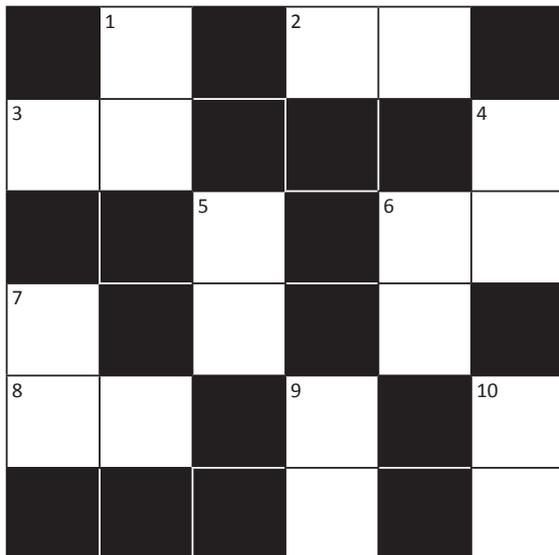
2×3

5×6

6×2

1×6

5 Complete the cross number puzzle:



Across

2. 9×3

3. 3×6

6. 5×6

8. 7×6

Down

1. 8×6

4. 10×6

5. 9×6

6. 6×6

7. 4×6

9. 6×3

10. 7×3

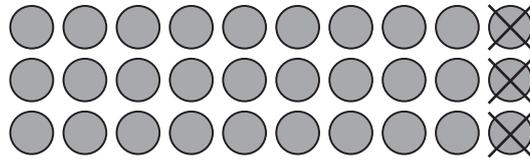
6 What number am I? I am in the 3 times table, 4 times table and 6 times table. I'm not 12.

I am

Using known facts – 9 times table

If you get stuck on a 9 times table fact, you can use the 10 times table facts and then build down.

$$3 \times 9 = \boxed{?}$$



$$3 \times 10 = 30 - 3 \longrightarrow \text{So, } 3 \times 9 = 27$$

1 Think of the $\times 10$ facts and build down to get the $\times 9$ facts. The first one is done for you.

$\times 10$ table	Build down by	$\times 9$ table
$1 \times 10 = 10$	1	$1 \times 9 = 9$
$2 \times 10 = 20$		
$3 \times 10 = 30$		
$4 \times 10 = 40$		
$5 \times 10 = 50$		
$6 \times 10 = 60$		
$7 \times 10 = 70$		
$8 \times 10 = 80$		
$9 \times 10 = 90$		
$10 \times 10 = 100$		

2 Complete the $\times 9$:

\times	2	6	4	8	3	9	10	5	7
9									

Using known facts – 7 times table

If you get stuck on a 7 times table fact, remember the 8 times table fact and build down.

1 Think of the $\times 8$ table fact and build down to get the $\times 7$ table fact.

$\times 8$ table	Build down by	$\times 7$ table
$1 \times 8 = 8$	1	$1 \times 7 =$
$2 \times 8 = 16$	2	$2 \times 7 =$
$3 \times 8 = 24$	3	$3 \times 7 =$
$4 \times 8 = 32$		$4 \times 7 =$
$5 \times 8 = 40$		$5 \times 7 =$
$6 \times 8 = 48$		$6 \times 7 =$
$7 \times 8 = 56$		$7 \times 7 =$
$8 \times 8 = 64$		$8 \times 7 =$
$9 \times 8 = 72$		$9 \times 7 =$
$10 \times 8 = 80$		$10 \times 7 =$

2 Add the missing numbers to each fact:

a $\times 7 = 28$

b $\times 7 = 35$

c $\times 7 = 21$

d $\times 7 = 42$

e $\times 7 = 49$

f $\times 7 = 14$

3 Use the $\times 8$ to complete the $\times 7$:

\times	4	2	6	1	9	5	3	7	8
8									
7									

Using known facts – square numbers

A square number is a number multiplied by itself.

$$1 \times 1 = 1$$

$$1^2 = 1$$

$$2 \times 2 = 4$$

$$2^2 = 4$$

$$3 \times 3 = 9$$

$$3^2 = 9$$

1 Show these square numbers on the grid and write what they are equal to:

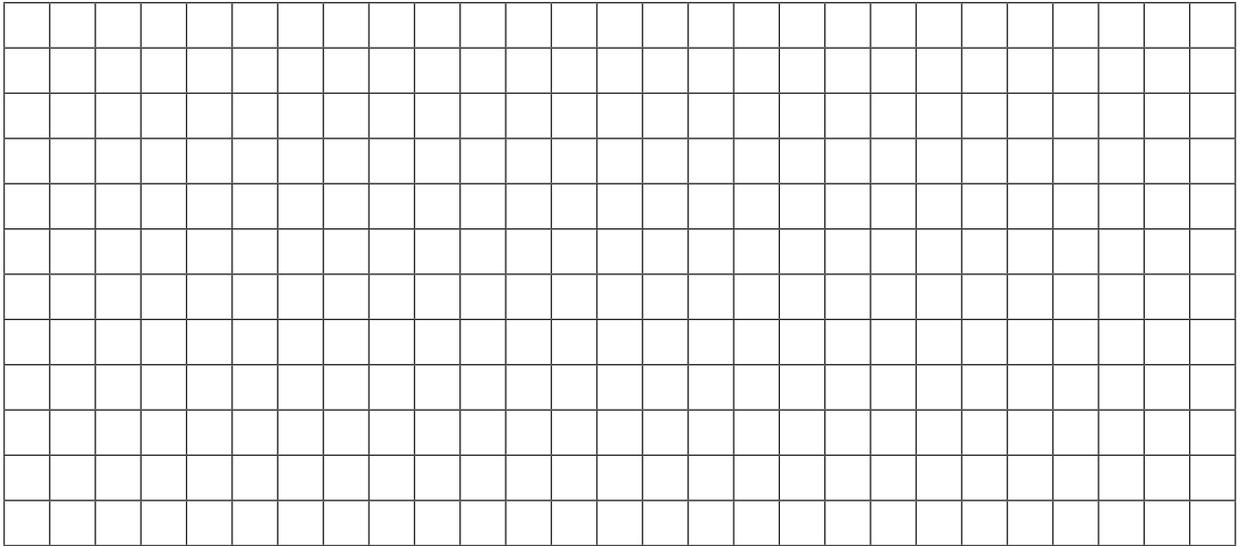
a $4^2 =$

b $6^2 =$

c $5^2 =$

d $3^2 =$

e $7^2 =$



2 Shade the square numbers on this multiplication grid:

×	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Using known facts – factors and multiples

When 2 numbers are multiplied together, the answer is called a multiple.
The first 3 multiples of 2 are 2, 4, 6.

$$1 \times 2 = 2$$

$$2 \times 2 = 4$$

$$3 \times 2 = 6$$

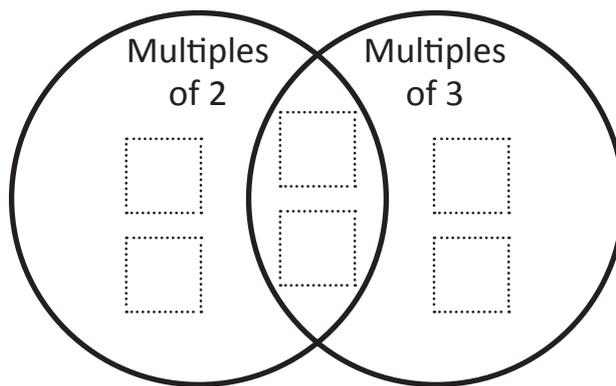
5, 10, 15, 20, 25, 30, 35, 40, 45, 50 are the first 10 multiples of 5.

1 List the first ten multiples of each number:

a	6	6									
b	2										
c	10										
d	3										
e	4										

2 Write these numbers in the correct spots on the Venn diagram:

8 4 9 6 12 3



The space in the diagram where the circles overlap is where you put numbers that are *both* multiples of 2 and 3.

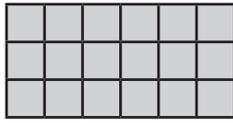


THINK

3 Can you think of any other numbers up to 60 that could go into the overlapping space in the Venn diagram above?

Using known facts – factors and multiples

Factors are numbers that you multiply together to give a multiple.



$$3 \times 6 = 18$$

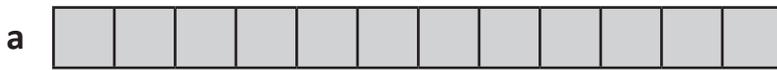


$$2 \times 9 = 18$$

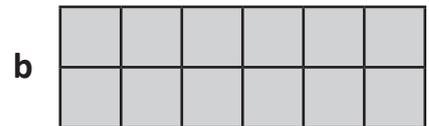
These arrays show some of the factors of 18: 3, 6, 2 and 9.

Can you think of any other factors of 18?

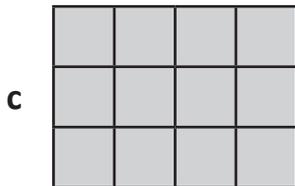
1 Complete the number sentence for each set of arrays and then list the factors.



$$\square \times \square = \square$$



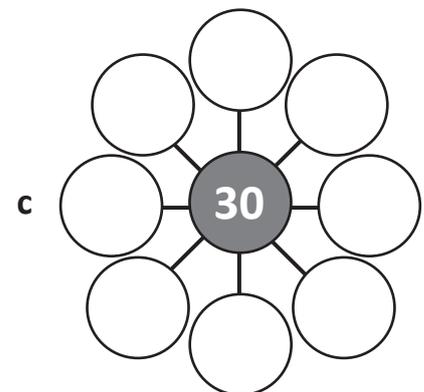
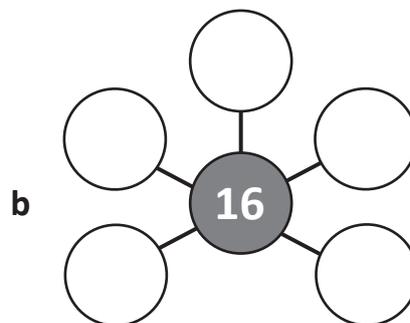
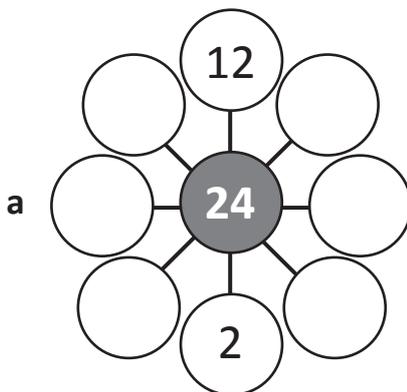
$$\square \times \square = \square$$



$$\square \times \square = \square$$

d The factors of 12 are:

2 Complete each diagram to show the factors of the number in the middle circle:



Mental multiplication strategies – multiplying by 10 and 100

When we multiply any number by 10, a zero goes in the units column and the digits all move one space along to the left.

When we multiply any number by 100, a zero goes in both the units and the tens columns and all the digits move two spaces along to the left.

Thousands	Hundreds	Tens	Units
		4	5
	4	5	0
4	5	0	0

× 10
× 100

1 Use the place value tables to multiply these numbers by 10 and 100:

a

Th	H	T	U
		1	5

× 10
× 100

b

Th	H	T	U
		4	8

× 10
× 100

c

Th	H	T	U
		7	2

× 10
× 100



Can you see a pattern in each of the tables?

2 Use patterns to solve these:

a $14 \times 1 =$

$14 \times 10 =$

$14 \times 100 =$

b $25 \times 1 =$

$25 \times 10 =$

$25 \times 100 =$

c $82 \times 1 =$

$82 \times 10 =$

$82 \times 100 =$

Mental multiplication strategies – multiplying by 10 and 100

How do you multiply by other multiples of 10? Let's look at 8×20 .
We can use known times tables facts and write this as place value amounts:

$$8 \times 2 \text{ tens} = 16 \text{ tens So, } 8 \times 20 = 160$$

1 Draw lines from the numbers written as place value amounts to the times tables facts:

10 tens 14 tens 36 tens 27 tens 12 tens 16 tens

3×4 tens 4×4 tens 5×2 tens 7×2 tens 6×6 tens 9×3 tens

2 Write the digit that represents each place value amount:

- | | | | | | |
|---|--------------------------------|---|--------------------------------|---|--------------------------------|
| a | 10 tens = <input type="text"/> | b | 36 tens = <input type="text"/> | c | 12 tens = <input type="text"/> |
| d | 15 tens = <input type="text"/> | e | 22 tens = <input type="text"/> | f | 8 tens = <input type="text"/> |
| g | 19 tens = <input type="text"/> | h | 16 tens = <input type="text"/> | i | 18 tens = <input type="text"/> |

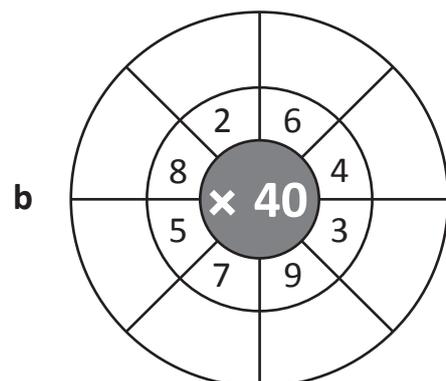
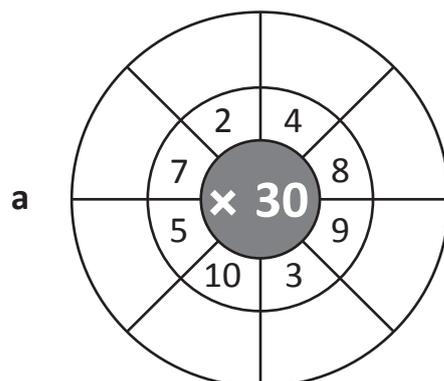
3 First complete the hints and then use them to write the facts:

Hints:

Facts:

- | | | |
|---|---|--------------------------------------|
| a | 4×6 tens = <input type="text"/> tens | $4 \times 60 =$ <input type="text"/> |
| b | 9×2 tens = <input type="text"/> tens | $9 \times 20 =$ <input type="text"/> |
| c | 2×7 tens = <input type="text"/> tens | $2 \times 70 =$ <input type="text"/> |

4 Complete the number wheels:



Mental multiplication strategies – doubling strategy

There are many double facts that you should know.
This includes numbers outside the times tables we have been working on.
Here are 2 double facts that are handy to know:

double 15 is 30 double 50 is 100 Can you think of more?

1 Complete these function machines:

a

Double	
IN	OUT
15	30
24	
30	
45	
18	

b

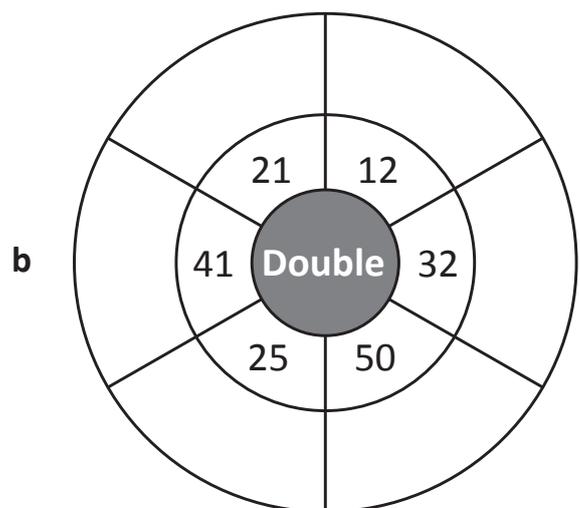
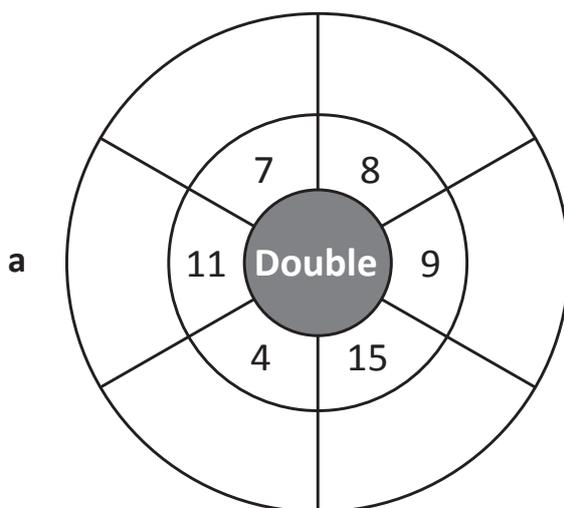
Double-double	
IN	OUT
15	60
24	
30	
45	
50	

Can you see what double-double is the same as? Yes, that's right, it's the same as $\times 4$.



REMEMBER

2 Complete these doubling wheels:



Mental multiplication strategies – doubling strategy

We also use doubling when we multiply by 4 and by 8.

To multiply a number by 4,
double it twice.

$10 \times 4 = 40$	
Double 10 once	20
Double 10 twice	40

To multiply a number by 8,
double it 3 times.

$11 \times 8 = 88$	
Double 11 once	22
Double 11 twice	44
Double 11 three times	88

1 Keep doubling to get the $\times 4$ and $\times 8$ facts. Here are some tables to help you. The first one has been done for you.

a

$12 \times 4 = 48$	
Double 12 once	24
Double 12 twice	48

b

$15 \times 4 =$ <input type="text"/>	
Double 15 once	
Double 15 twice	

c

$18 \times 4 =$ <input type="text"/>	
Double 18 once	
Double 18 twice	

d

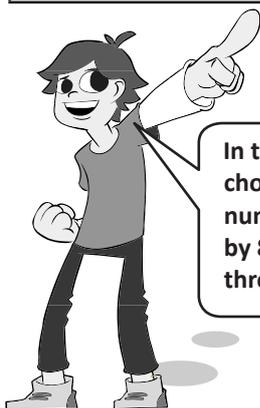
$22 \times 4 =$ <input type="text"/>	
Double 22 once	
Double 22 twice	

e

$16 \times 8 =$ <input type="text"/>	
Double 16 once	
Double 16 twice	
Double 16 three times	

f

$35 \times 8 =$ <input type="text"/>	
Double 35 once	
Double 35 twice	
Double 35 three times	



In this last table choose a 2-digit number to multiply by 8 and double it three times.

g

<input type="text"/> $\times 8 =$ <input type="text"/>	
Double <input type="text"/> once	
Double <input type="text"/> twice	
Double <input type="text"/> three times	

Mental multiplication strategies – split strategy

The split strategy is when we multiply numbers in 2 pairs and then add the parts. Let's use the split strategy for 26×4 .

- Split 26 into 20 and 6.
- Multiply each part.
- Add the answers together.

$$26 \times 4 \longrightarrow 20 \times 4 + 6 \times 4$$

$$80 + 24 = 104$$

$$\text{So, } 26 \times 4 = 104$$

1 Use the split strategy to answer these:

a $34 \times 3 \longrightarrow 30 \times 3 + 4 \times 3$

$$90 + \boxed{} = \boxed{}$$

$$\text{So, } 34 \times 3 = \boxed{}$$

b $45 \times 5 \longrightarrow \boxed{} \times \boxed{} + \boxed{} \times \boxed{}$

$$\boxed{} + \boxed{} = \boxed{}$$

$$\text{So, } 45 \times 5 = \boxed{}$$

c $52 \times 4 \longrightarrow \boxed{} \times \boxed{} + \boxed{} \times \boxed{}$

$$\boxed{} + \boxed{} = \boxed{}$$

$$\text{So, } 52 \times 4 = \boxed{}$$

Mental multiplication strategies – compensation

Use the compensation strategy to make it easier to multiply 2-digit numbers that are close to a ten.

Look at 4×19 .

19 is close to 20, so we can multiply by the next multiple of ten which is 20. Then we build down because we have an extra group of 4.

$$4 \times 19 \longrightarrow 4 \times 20 = 80 - 4$$

$$\text{So, } 19 \times 4 = 76$$

1 Use the compensation strategy to answer these:

a $5 \times 29 \longrightarrow 5 \times \square = \square - \square$

So, $5 \times 29 = \square$

b $3 \times 49 \longrightarrow 3 \times \square = \square - \square$

So, $3 \times 49 = \square$

c $4 \times 39 \longrightarrow 4 \times \square = \square - \square$

So, $4 \times 39 = \square$

2 Use the compensation strategy to answer these questions. This time you need to look for more than one extra group to subtract:

a $4 \times 18 \longrightarrow 4 \times \square = \square - \square$

So, $4 \times 18 = \square$

b $3 \times 17 \longrightarrow 3 \times \square = \square - \square$

So, $3 \times 17 = \square$

We have rounded up to 20. So instead of 4×18 we have 4×20 . This is 2 more groups of 4. So we subtract 8.



THINK

Mental multiplication strategies – choose a strategy



- 1 Roll a die to get the missing number, then use either the split or compensation strategy to get the answer. You can place the numbers rolled on the die in any question.

a $25 \times \square \rightarrow$

So, $25 \times \square = \square$

b $36 \times \square \rightarrow$

So, $36 \times \square = \square$

c $49 \times \square \rightarrow$

So, $49 \times \square = \square$

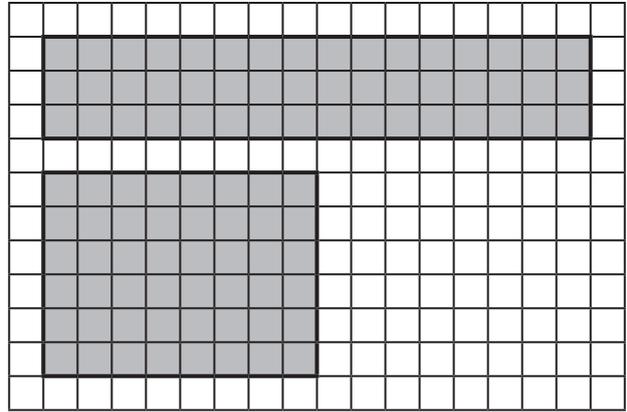
d $58 \times \square \rightarrow$

So, $58 \times \square = \square$

Mental multiplication strategies – doubling and halving

We can change the factors of a multiplication question to make it easier. Look at 16×3 . If we halve the larger factor and double the smaller factor, we make an array on the grid that is the same size. Both arrays have the same amount of squares. Count the squares, are they equal to 8×6 ?

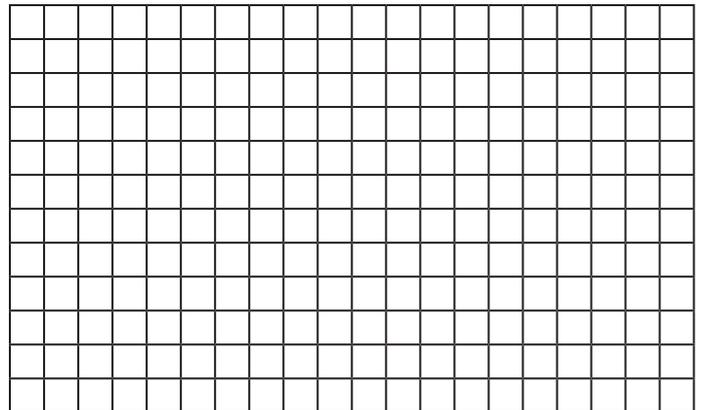
$$\begin{array}{ccc}
 16 & \times & 3 \\
 \downarrow & & \downarrow \\
 \boxed{\text{Halve}} & & \boxed{\text{Double}} \\
 8 & \times & 6 = 48
 \end{array}$$



1 Make these problems easier by using doubling and halving. Shade an array for each:

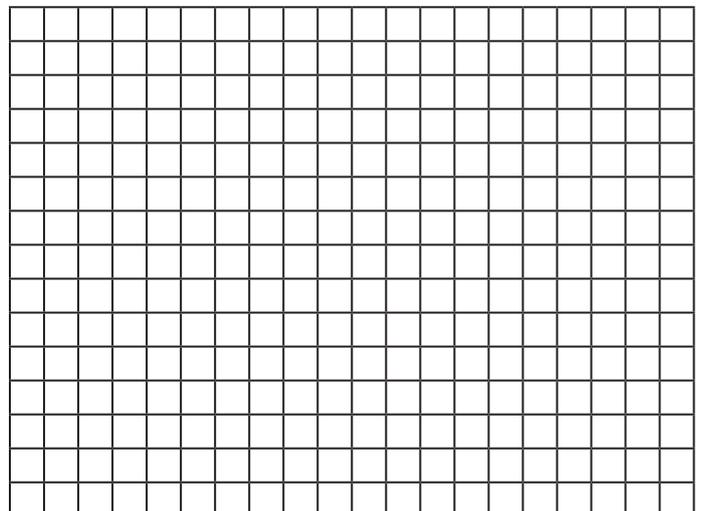
a

$$\begin{array}{ccc}
 18 & \times & 3 \\
 \downarrow & & \downarrow \\
 \boxed{\text{Halve}} & & \boxed{\text{Double}} \\
 \boxed{} & \times & \boxed{} = \boxed{}
 \end{array}$$



b

$$\begin{array}{ccc}
 14 & \times & 4 \\
 \downarrow & & \downarrow \\
 \boxed{\text{Halve}} & & \boxed{\text{Double}} \\
 \boxed{} & \times & \boxed{} = \boxed{}
 \end{array}$$



Mental multiplication strategies – doubling and halving

2 Use the doubling and halving strategy to solve these:

a 14 × 3

↓ ↓

Halve

Double

□ × □ = □

b 48 × 5

↓ ↓

Halve

Double

□ × □ = □

c 16 × 5

↓ ↓

Halve

Double

□ × □ = □

d 64 × 5

↓ ↓

Halve

Double

□ × □ = □

3 Follow this doubling and halving trail through to the bottom:

a Halve Double

8 × 56 = □ ?

↓ ↓

□

×

□

↓ ↓

□

×

□

↓ ↓

□

×

□

So, 8 × 56 = □

b Halve Double

8 × 35 = □ ?

↓ ↓

□

×

□

↓ ↓

□

×

□

↓ ↓

□

×

□

So, 8 × 35 = □

c Halve Double

8 × 45 = □ ?

↓ ↓

□

×

□

↓ ↓

□

×

□

↓ ↓

□

×

□

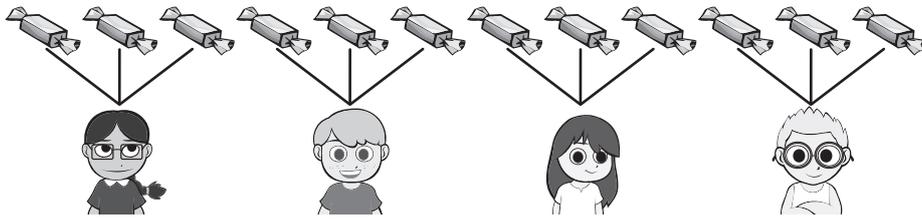
So, 8 × 45 = □

d What do you notice?

Division – division is sharing and grouping

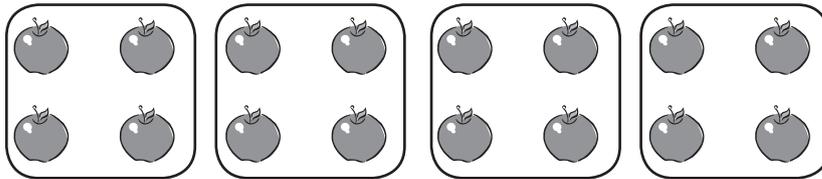
Division can mean sharing *or* grouping.

There are 12 lollies shared between 4 kids. How many are **in** each share?



$$12 \div 4 = 3$$

There are 16 apples and 4 go into each basket. How many baskets do I need?



$$16 \div 4 = 4$$

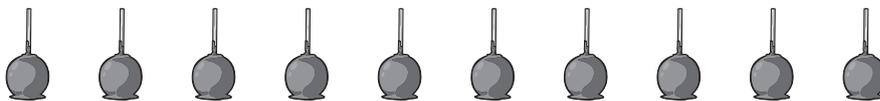
1 Solve these sharing and grouping questions:

a There are 9 cupcakes and 3 kids are sharing. How many are in each share?



$$\square \div \square = \square$$

b 10 lollies are shared between a group of kids so they each get 2. How many kids are sharing?



$$\square \div \square = \square$$

c There are 24 pencils and 6 pencil pots. How many pencils go into each pencil pot?



$$\square \div \square = \square$$

Division – division is sharing and grouping

- 2 Draw pictures to show these division questions. Then write the division fact and decide whether it is a sharing or a grouping question.

If you need to find out how many items there are in each share, it's a sharing question. If you need to find out the number of equal shares, it's a grouping question.



CHECK

- a Divide 16 lollies between 4 girls. How many does each girl get?

$$\square \div \square = \square$$

sharing / grouping

- b From a packet of 24 pencils, each person will get 6. How many people are sharing the pencils?

$$\square \div \square = \square$$

sharing / grouping

- c 48 eggs are laid by 6 hens. If they all laid the same amount, how many did each hen lay?

$$\square \div \square = \square$$

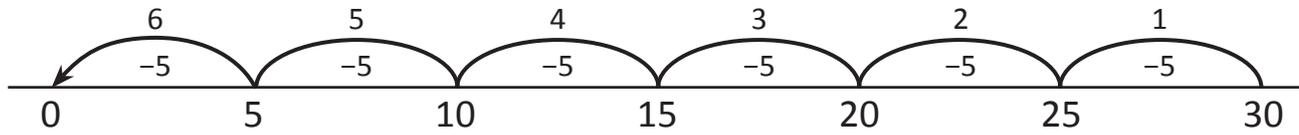
sharing / grouping

Division – division is repeated subtraction

Division can also be thought of as repeated subtraction.

Look at $30 \div 5 = \square$ This question is asking how many groups of 5 there are in 30.

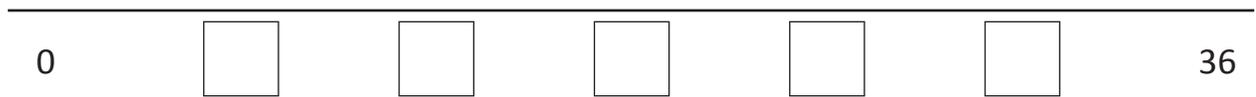
Jump in 5s along the number line and then count the jumps.



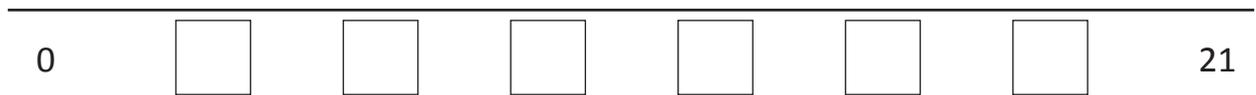
So, $30 \div 5 = 6$

- 1** Show these division facts as repeated subtraction. First label the number lines and then show the jumps.

a $36 \div 6 = \square$

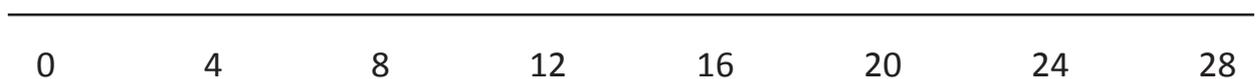


b $21 \div 3 = \square$

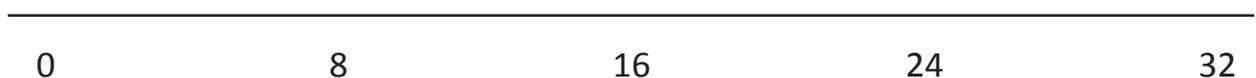


- 2** Write a division fact to match these number lines. Show the jumps.

a $\square \div \square = \square$

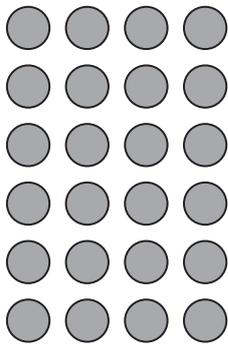


b $\square \div \square = \square$



Division – linking multiplication and division facts

Knowing multiplication facts will help with division facts. This is because they are opposites. Look at how we can describe this array:



$6 \times 4 = 24$

6 groups of 4 is 24.

$4 \times 6 = 24$

4 groups of 6 is 24.

$24 \div 4 = 6$

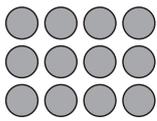
24 divided into 4 shares is 6.

$24 \div 6 = 4$

24 divided into 6 shares is 4.

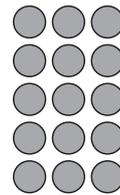
1 Describe each of these arrays using two multiplication and two division facts:

a



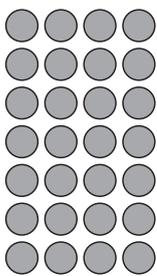
<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>

b



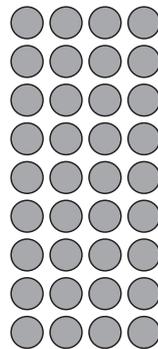
<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>

c



<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>

d



<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>

2 Draw an array of 6 rows of 3 then describe it with multiplication and division facts.

<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	÷	<input type="text"/>	=	<input type="text"/>

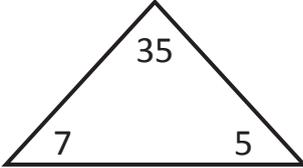
This is also called a fact family. ✨

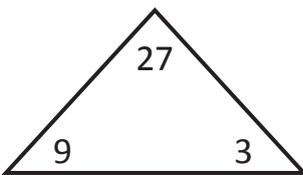


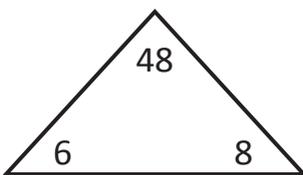
REMEMBER

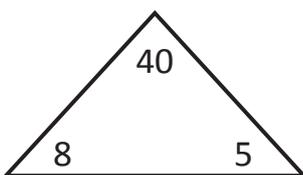
Division – linking multiplication and division facts

3 Write a fact family for each set of numbers in the triangle. The first one has been done for you.

a $\boxed{5} \times \boxed{7} = \boxed{35}$  $\boxed{35} \div \boxed{5} = \boxed{7}$
 $\boxed{7} \times \boxed{5} = \boxed{35}$ $\boxed{35} \div \boxed{7} = \boxed{5}$

b $\boxed{} \times \boxed{} = \boxed{}$  $\boxed{} \div \boxed{} = \boxed{}$
 $\boxed{} \times \boxed{} = \boxed{}$ $\boxed{} \div \boxed{} = \boxed{}$

c $\boxed{} \times \boxed{} = \boxed{}$  $\boxed{} \div \boxed{} = \boxed{}$
 $\boxed{} \times \boxed{} = \boxed{}$ $\boxed{} \div \boxed{} = \boxed{}$

d $\boxed{} \times \boxed{} = \boxed{}$  $\boxed{} \div \boxed{} = \boxed{}$
 $\boxed{} \times \boxed{} = \boxed{}$ $\boxed{} \div \boxed{} = \boxed{}$

4 For these problems, think of a multiplication fact to help write the division fact:

a £25 is shared between 5 people. How much does each person get?

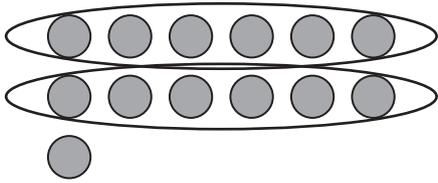
$\boxed{} \times \boxed{} = \boxed{}$ $\boxed{} \div \boxed{} = \boxed{}$

b 45 people get into 9 cars. How many people are in each car?

$\boxed{} \times \boxed{} = \boxed{}$ $\boxed{} \div \boxed{} = \boxed{}$

Division – remainders

Sometimes division is not exact.

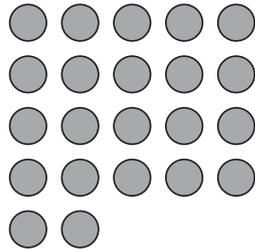


From 13, we can make 2 fair shares of 6 with 1 left over. We call the left over the remainder.

$$13 \div 6 = 2 \text{ remainder } 1$$

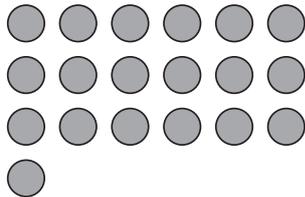
1 In each array, ring the fair shares to see the remainder:

a



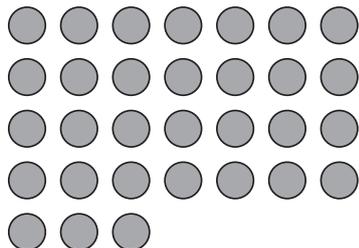
$$22 \div 5 = \square \text{ remainder } \square$$

b



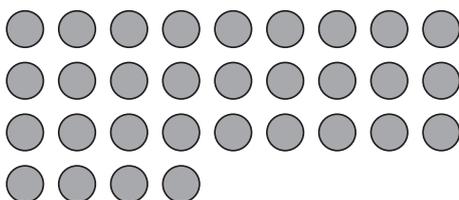
$$19 \div 6 = \square \text{ remainder } \square$$

c



$$31 \div 7 = \square \text{ remainder } \square$$

d



$$31 \div 9 = \square \text{ remainder } \square$$

Division – remainders

Now use your multiplication facts.

$$25 \div 6 = \boxed{\quad ? \quad} \quad \text{Think} \quad 4 \times 6 = 24 + 1 \text{ is } 25$$

$$\text{So, } 25 \div 6 = 4 \text{ remainder } 1$$

2 Use your multiplication facts to write the division facts and the remainder:

a $32 \div 10 = \boxed{\quad ? \quad}$ Think $\boxed{\quad} \times \boxed{\quad} = \boxed{\quad} + \boxed{\quad}$ is $\boxed{\quad}$

So, $\boxed{\quad} \div \boxed{\quad} = \boxed{\quad}$ remainder $\boxed{\quad}$

b $30 \div 4 = \boxed{\quad ? \quad}$ Think $\boxed{\quad} \times \boxed{\quad} = \boxed{\quad} + \boxed{\quad}$ is $\boxed{\quad}$

So, $\boxed{\quad} \div \boxed{\quad} = \boxed{\quad}$ remainder $\boxed{\quad}$

c $37 \div 9 = \boxed{\quad ? \quad}$ Think $\boxed{\quad} \times \boxed{\quad} = \boxed{\quad} + \boxed{\quad}$ is $\boxed{\quad}$

So, $\boxed{\quad} \div \boxed{\quad} = \boxed{\quad}$ remainder $\boxed{\quad}$

3 Complete each word problem:

a 39 pencils were shared between 6 kids. How many did each kid get?

$$\boxed{\quad} \div \boxed{\quad} = \boxed{\quad} \text{ remainder } \boxed{\quad}$$

b 43 fish were divided between 6 tanks. How many fish are in each tank?

$$\boxed{\quad} \div \boxed{\quad} = \boxed{\quad} \text{ remainder } \boxed{\quad}$$

c From 17 flowers, 5 flowers were arranged in each vase. How many vases were used?

$$\boxed{\quad} \div \boxed{\quad} = \boxed{\quad} \text{ remainder } \boxed{\quad}$$

4 Write in the missing digit to make this statement true:

$$\boxed{\quad} \div 6 = 8 \text{ remainder } 2$$

Mental division strategies – dividing by 10 and 100

When we divide any number by 10, we move the number one place value space to the right.

When we divide any number by 100, we move the number two place value spaces to the right.

Thousands	Hundreds	Tens	Units	
6	7	0	0	
	6	7	0	÷ 10
		6	7	÷ 100

1 Use the place value tables to divide these numbers by 10 and 100.

a

Th	H	T	U	
5	3	0	0	
				÷ 10
				÷ 100

b

Th	H	T	U	
4	1	0	0	
				÷ 10
				÷ 100

c

Th	H	T	U	
8	4	0	0	
				÷ 10
				÷ 100

d

Th	H	T	U	
2	4	0	0	
				÷ 10
				÷ 100

2 Use patterns to solve these:

a $1\ 400 \div 1 = \square$ $1\ 400 \div 10 = \square$ $1\ 400 \div 100 = \square$

b $5\ 600 \div 1 = \square$ $5\ 600 \div 10 = \square$ $5\ 600 \div 100 = \square$

c $3\ 500 \div 1 = \square$ $3\ 500 \div 10 = \square$ $3\ 500 \div 100 = \square$

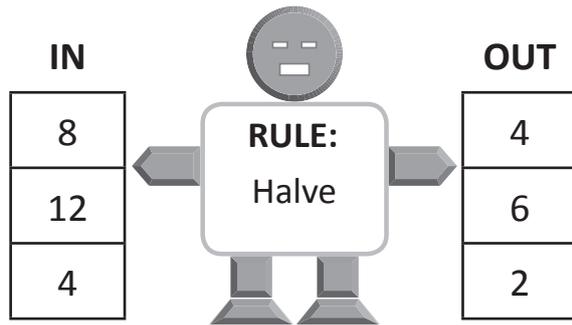
3 Use a calculator to solve these:

a $270 \div 100 = \square$

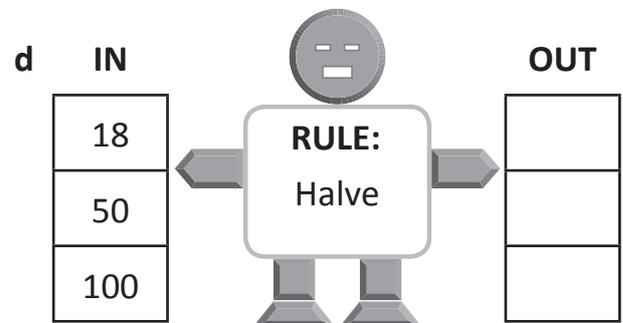
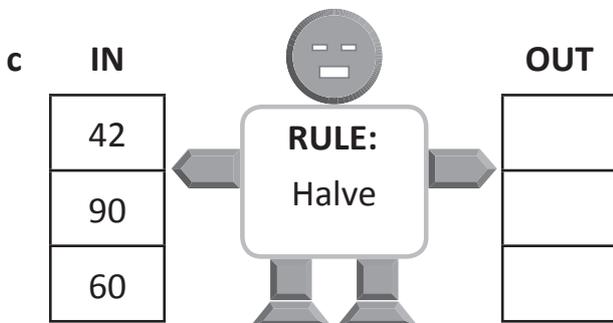
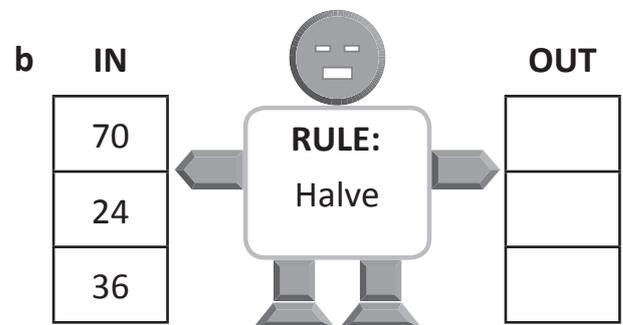
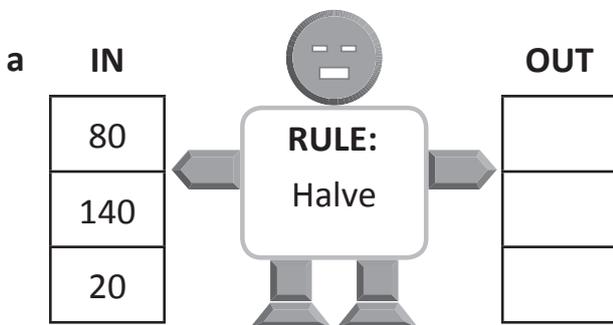
b $49 \div 10 = \square$

Mental division strategies – halving strategy

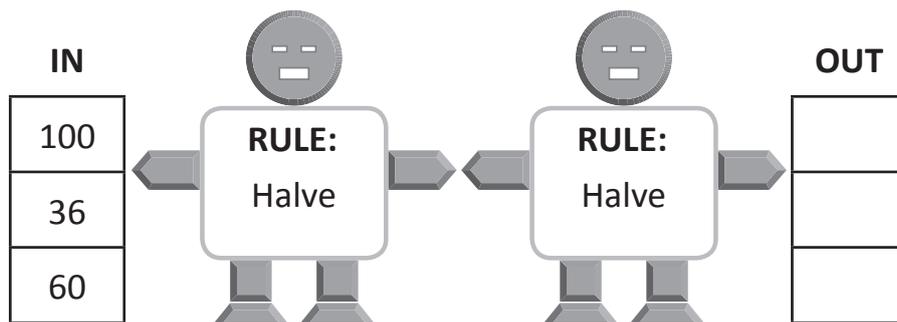
When you halve numbers you are dividing them by 2. In this function machine, numbers go IN, have the rule applied and come OUT again.



- 1 Complete the halving function machines. Halve the number going IN the machine and write the answer in the OUT column:

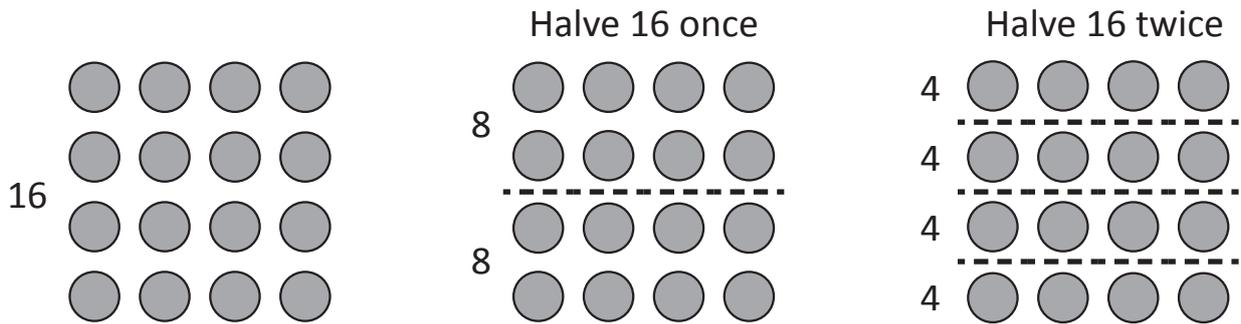


- 2 Below is a halving-halving function machine. The number goes IN and is halved and then halved again and comes OUT.



Mental division strategies – halving strategy

We also use halving-halving to divide by 4. Look at these diagrams:



3 Use the tables for halving-halving to divide by 4:

a

$80 \div 4 =$	<input type="text"/>
Halve 80 once	<input type="text"/>
Halve 80 twice	<input type="text"/>

b

$48 \div 4 =$	<input type="text"/>
Halve 48 once	<input type="text"/>
Halve 48 twice	<input type="text"/>

c

$64 \div 4 =$	<input type="text"/>
Halve 64 once	<input type="text"/>
Halve 64 twice	<input type="text"/>

d

$120 \div 4 =$	<input type="text"/>
Halve 120 once	<input type="text"/>
Halve 120 twice	<input type="text"/>

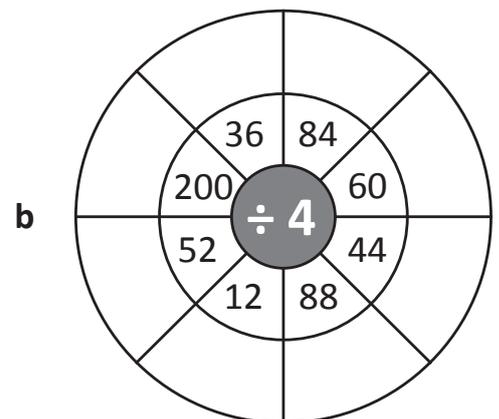
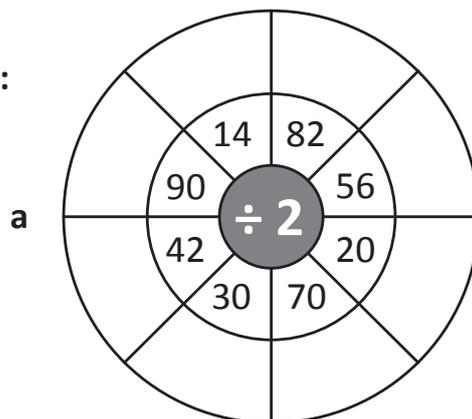
e

$244 \div 4 =$	<input type="text"/>
Halve 244 once	<input type="text"/>
Halve 244 twice	<input type="text"/>

f

$88 \div 4 =$	<input type="text"/>
Halve 88 once	<input type="text"/>
Halve 88 twice	<input type="text"/>

4 Complete the division wheels:



Mental division strategies – split strategy

Division problems can be much easier to solve if you split the number.

Look at $125 \div 5$.

Can we split the number into two multiples of 5?

Yes, we can split 125 into 100 and 25.

We divide each part by 5 and then add the two answers together.

$$\begin{array}{r} 125 \div 5 \\ \swarrow \downarrow \\ 100 \quad 25 \\ \div 5 \quad \div 5 \\ 20 + 5 = 25 \end{array}$$

1 Use the split strategy to divide these by 5:

a $115 \div 5$

$\div 5$ $\div 5$

+ =

b $135 \div 5$

$\div 5$ $\div 5$

+ =

2 Use the split strategy to divide these by 4:

a $64 \div 4$

$\div 4$ $\div 4$

+ =

b $116 \div 4$

$\div 4$ $\div 4$

+ =

3 Use the split strategy to divide these by 3:

a $330 \div 3$

$\div 3$ $\div 3$

+ =

b $612 \div 3$

$\div 3$ $\div 3$

+ =

Mental division strategies – strategy review

Review your division strategies.

- 1 Use either the halving strategy or the split strategy to complete the tables. The first one has been done for you.

a Use the split strategy:

$$48 \div 3 = \boxed{16}$$

48 is 30 + 18
30 ÷ 3 = 10 and 18 ÷ 3 = 6
10 + 6 = 16

b Use the halving strategy:

$$64 \div 4 = \boxed{}$$

c Use the split strategy:

$$312 \div 3 = \boxed{}$$

d Use the halving strategy:

$$140 \div 4 = \boxed{}$$

- 2 Solve this riddle by matching the letter to the answer. Use a mental division strategy for each problem.

What is it that the more you take, the more you leave behind?

$$68 \div 4 = \boxed{} \quad \boxed{} \text{ s}$$

$$90 \div 6 = \boxed{} \quad \boxed{} \text{ p}$$

$$135 \div 5 = \boxed{} \quad \boxed{} \text{ e}$$

$$1\,200 \div 10 = \boxed{} \quad \boxed{} \text{ f}$$

$$240 \div 4 = \boxed{} \quad \boxed{} \text{ o}$$

$$128 \div 4 = \boxed{} \quad \boxed{} \text{ t}$$

120	60	60	32	17	32	27	15	17

Written methods – contracted multiplication

	H	T	U
		15	4
x			3
	1	6	2

Start with the units. $4 \times 3 = 12$ units.

Rename this as 1 ten and 2 units. Put the 2 in the units column and regroup the 1 to the tens column.

3×5 plus the regrouped 1 is 16 tens.

Rename this as 1 hundred and 6 tens.

1 Practise these problems:

a

	H	T	U
			42
x			9

b

	H	T	U
			38
x			7

c

	H	T	U
			25
x			4

d

	H	T	U
			26
x			4

e

	H	T	U
			55
x			8

f

	H	T	U
			62
x			7

2 Use contracted multiplication to solve these word problems:

a On a farm, 6 lambs were born every day over 25 days. How many lambs were born in total?

	H	T	U
x			

b For my school fete day, I baked 9 trays of cupcakes. If there are 14 cupcakes on each tray, how many did I bake in total?

	H	T	U
x			

Written methods – extended multiplication

	H	T	U
		3	4
×			3
		1	2
		9	0
	1	0	2

In extended multiplication, we multiply the units and tens separately, then add the answers together.

1 Practise these problems:

a

	H	T	U
		2	3
×			4

← (4 × 3)

← (4 × 20)

b

	H	T	U
		3	6
×			5

← (5 × 6)

← (5 × 30)

c

	H	T	U
		7	4
×			6

← (___ × ___)

← (___ × ___)

d

	H	T	U
		5	2
×			7

← (___ × ___)

← (___ × ___)

2 Use extended multiplication to solve this word problem:

In a pet store, there are 7 tanks of tropical fish with 14 fish per tank.

How many fish are there altogether?

	H	T	U
×			

← (___ × ___)

← (___ × ___)

Written methods – short division

Another way to represent division is with the division symbol.

	T	U
		6
		6
6	3	6

This is the same as $36 \div 6 = 6$

If the answer is a single digit, it should go in the units column.

1 Solve these division problems using the division symbol:

a

	□	
	5	
	5	
5	3	5

b

	□	
	8	
	8	
4	2	8

c

	□	
	8	
	8	
9	1	8

d

	□	
	4	
	4	
6	5	4

e

	□	
	4	
	4	
2	1	4

f

	□	
	6	
	6	
4	1	6

g

	□	
	5	
	5	
5	2	5

h

	□	
	9	
	9	
7	4	9

i

	□	
	8	
	8	
8	4	8

2 Use the division symbol to solve each problem:

a 42 cupcakes were iced by 7 kids. If they each iced the same amount, how many did they ice each?

	□	
	□	
	□	
□	□	□

b How many pots were used if 6 seeds were planted in each pot from a packet of 54?

	□	
	□	
	□	
□	□	□

c I run the same distance each day. Over 9 days the total distance is 72 km. How far did I run each day?

	□	
	□	
	□	
□	□	□

Written methods – short division with 3-digit numbers

In short division with 3-digit numbers we split the number:

468 is $400 + 60 + 8$

400 divided by 2 is 200, so we put a 2 in the hundreds place.

60 divided by 2 is 30, so we put a 3 in the tens place.

8 is divided by 2 is 4, so we put a 4 in the units place.

H	T	U
2	3	4
2)	4 6 8

1 Practise splitting these:

a 368 is _____ + _____ + _____

b 445 is _____ + _____ + _____

c 567 is _____ + _____ + _____

d 235 is _____ + _____ + _____

2 Now put these split numbers back together:

a $500 + 70 + 8$ is _____

b $700 + 90 + 4$ is _____

c $200 + 40 + 6$ is _____

d $800 + 50 + 5$ is _____

3 Solve these division problems with 3-digit numbers:

a
$$\begin{array}{r} \square \square \square \\ 4 \overline{) 8 \ 4 \ 4} \end{array}$$

b
$$\begin{array}{r} \square \square \square \\ 3 \overline{) 6 \ 9 \ 3} \end{array}$$

c
$$\begin{array}{r} \square \square \square \\ 2 \overline{) 8 \ 4 \ 2} \end{array}$$

d
$$\begin{array}{r} \square \square \square \\ 2 \overline{) 4 \ 8 \ 8} \end{array}$$

4 Here are two division problems with missing numbers in the questions. Find out the missing numbers by using the numbers that are part of the answer as clues.

a
$$\begin{array}{r} \square \square \square \\ \square \overline{) 4 \ \square \ 4} \end{array}$$

b
$$\begin{array}{r} \square \square \square \\ 3 \overline{) \square \ 3 \ 6} \end{array}$$

Written methods – short division with 3-digit numbers

Sometimes we need to split the number a different way,

for example: $515 = 500 + 15$

500 divided by 5 is 100, so we put a 1 in the hundreds place.

15 divided by 5 is 3, so we put a 3 in the units place.

What goes in the tens place?

A zero does. The zero has the very important job of keeping the other numbers in their place!

H	T	U
1	0	3
5	5	15

5 Practise these problems. We have put the zero in to remind you:

a $4 \overline{) \begin{array}{|c|c|c|} \hline \square & 0 & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 8 & 1 & 2 \\ \hline \end{array}$

b $3 \overline{) \begin{array}{|c|c|c|} \hline \square & 0 & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 9 & 2 & 4 \\ \hline \end{array}$

c $3 \overline{) \begin{array}{|c|c|c|} \hline \square & 0 & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 9 & 1 & 2 \\ \hline \end{array}$

d $4 \overline{) \begin{array}{|c|c|c|} \hline \square & 0 & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 8 & 2 & 4 \\ \hline \end{array}$

6 Practise these problems. This time, you need to remember the zero!

a $3 \overline{) \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 9 & 1 & 8 \\ \hline \end{array}$

b $6 \overline{) \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 6 & 1 & 2 \\ \hline \end{array}$

c $4 \overline{) \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 8 & 3 & 2 \\ \hline \end{array}$

d $4 \overline{) \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array}} \begin{array}{|c|c|c|} \hline 8 & 1 & 6 \\ \hline \end{array}$



Getting ready

This is a game for 2 players. You will need a copy of this page, 6 counters each and 3 dice.



copy



What to do

Player 1 rolls all 3 dice and chooses 2 of the numbers to multiply. If the player can see the answer in the grid, they claim this number by placing a counter over the number. Then Player 2 has a turn. The winner is the first to place all 6 counters on the grid.

20	15	12	2	8
6	12	6	16	6
36	20	18	8	10
12	10	6	12	4
10	12	15	24	25



Getting ready

This is a game for three players. Each player needs a copy of this page. The caller needs a pile of the numbers from 1 to 9.



copy



What to do

Each multiplication grid contains all the answers, while the factors are missing. Remember factors are the numbers that you multiply to get the answer.

The aim of the game is to be the first player to fill their grid with the factors. One hint is provided in each grid to start you off. Choose one person to be the caller and the other two play the round. The caller picks a number without looking and reads it out to the players. The players write it on the grids, if it fits as a factor. The first to fill in one of the grids completely is the winner.

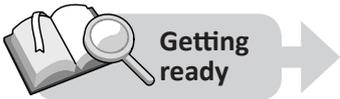
1	6
2	7
3	8
4	9
5	

×			
6	42	24	18
	63	36	27
	35	20	15

×	3		
	12	20	28
	18	30	42
	27	45	63

×			
	8	40	64
3	3	15	24
	9	45	72

×			9
	4	14	18
	2	7	9
	12	42	54



Getting ready

This is a game for two players. You will need a copy of page 43, a die and a pencil to write down your scores. You may like to make extra copies of page 43 to play again later.



copy



What to do

The aim of this game is to score the highest number of points each time without going over 20. Roll the dice and choose which strategy you will use. From the Strategy column, circle 1 for double, 2 for double-double or 3 for double-double-double. For example, Player 2 has rolled a 5 and has chosen strategy 3 double-double-double. This makes a score of 40 but because it is over 20 it doesn't count. Look at the rest of the sample game to see how the game turned out.

Strategy 1	Strategy 2	Strategy 3
Double	Double Double	Double Double Double

Sample game

Player 1		
Die	Strategy	Score
6	1	12
	2	
	3	
2	1	16
	2	
	3	
4	1	16
	2	
	3	
6	1	24
	2	
	3	
3	1	12
	2	
	3	
Total		56

Player 2		
Die	Strategy	Score
5	1	40
	2	
	3	
3	1	12
	2	
	3	
1	1	8
	2	
	3	
4	1	16
	2	
	3	
2	1	16
	2	
	3	
Total		52

Strategy 1	Strategy 2	Strategy 3
Double	Double Double	Double Double Double

Player 1		
Die	Strategy	Score
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
Total		

Player 2		
Die	Strategy	Score
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
	1	
	2	
	3	
Total		



What to do

Can you work out the value of each symbol?

The values are 2, 3, 4, 6, 8, 9 and 12. Remember, the same symbol means that it's the same number.

$$\diamond \times \diamond = \star \quad \square \times \square = \square$$

$$\diamond \times \diamond \times \diamond = \star \quad \square \times \square \times \square = \square$$

$$\diamond \times \star = \star \quad \square \times \square = \square$$

$$\nabla \times \star = \bigcirc \quad \square \times \square = \square$$

$$\nabla \times \nabla = \bigcirc \quad \square \times \square = \square$$

$$\nabla \times \diamond = \blacksquare \quad \square \times \square = \square$$

$$\blacksquare \times \diamond = \bigcirc \quad \square \times \square = \square$$

$\diamond = \square$	$\star = \square$	$\star = \square$	$\nabla = \square$
$\bigcirc = \square$	$\blacksquare = \square$	$\bigcirc = \square$	