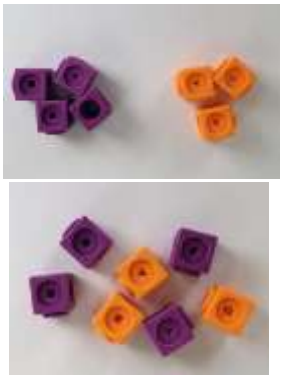
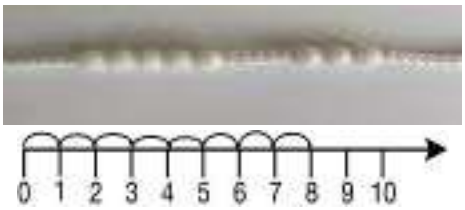
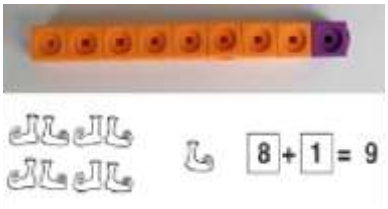
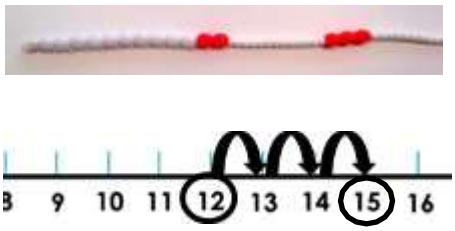
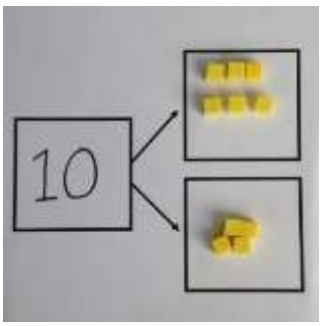
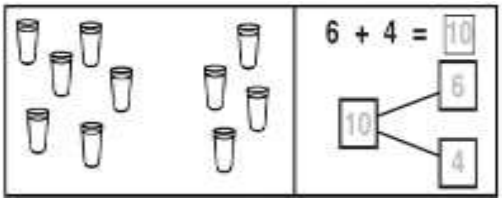
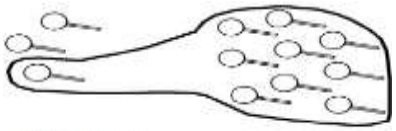
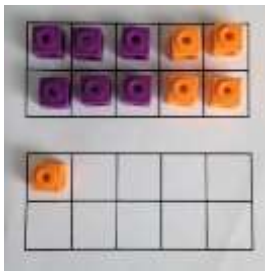
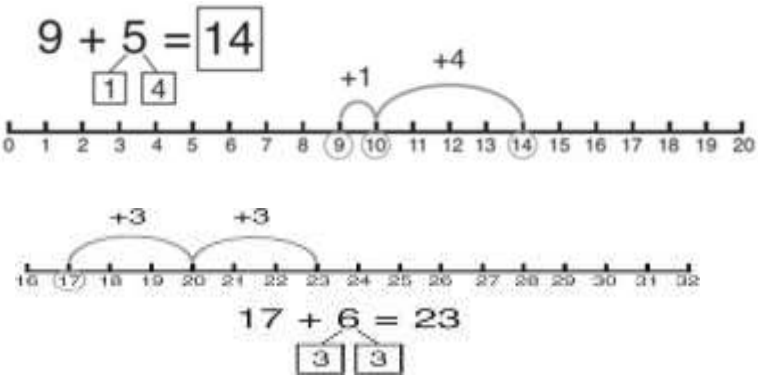
Progression in calculations

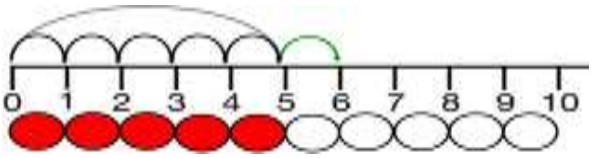
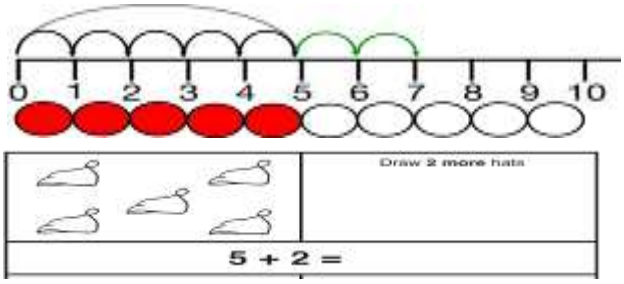
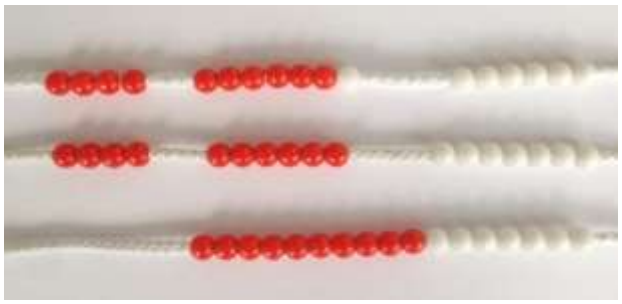
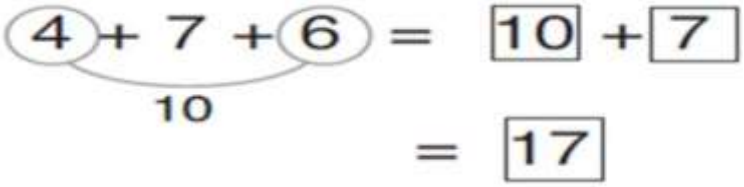
Year 1 – Year 6

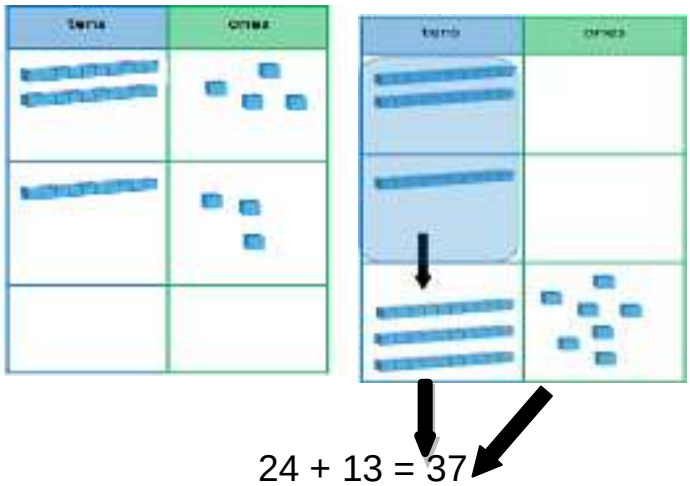
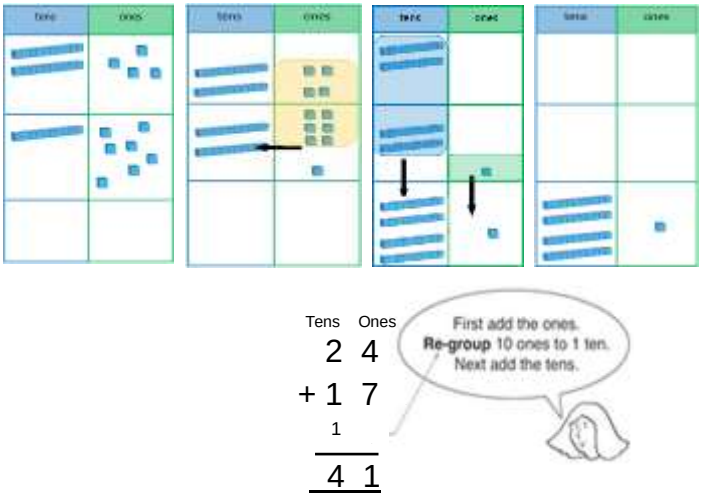
Progression in calculations Year 1

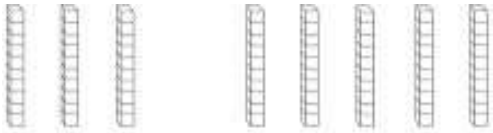
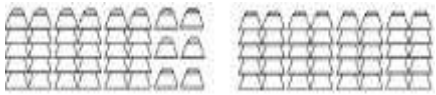
Y1 Addition

Strategy & guidance	CPA
Count all *Joining two groups and then recounting all objects using one-to-one correspondence*	3 + 4 = 7   5 + 3 = 8
Counting on *As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the greater number is more efficient.*	8 + 1 = 9  15 = 12 + 3 
Part-part-whole *Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse relationship between them.* *This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.*	  10 = 6 + 4 10 − 6 = 4 10 − 4 = 6 10 = 4 + 6

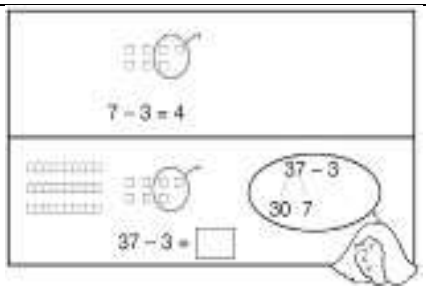
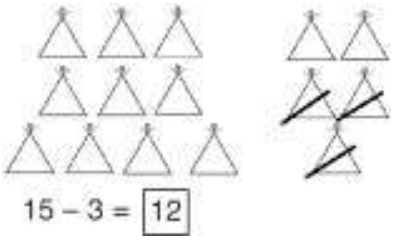
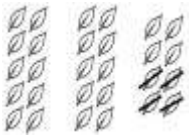
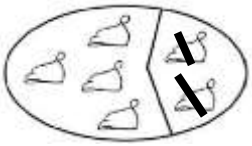
Regrouping ten ones to make ten *This is an essential skill that will support column addition later on.*	 3 + 9 =    3 + 9 = 12
'Make ten' strategy *Pupils should be encouraged to start at the greater number and partition the smaller number to make ten.* *The colours of the beads on the bead string make it clear how many more need to be added to make ten.* *Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.*	6 + 5 = 11  4 + 9 = 13  

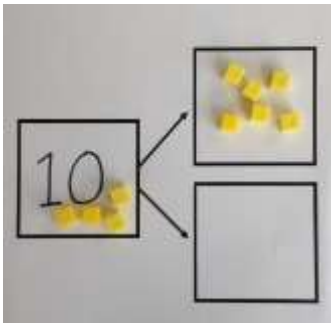
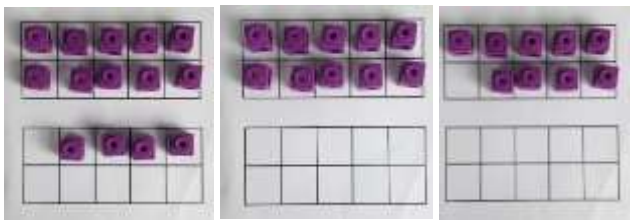
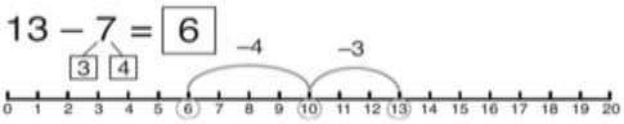
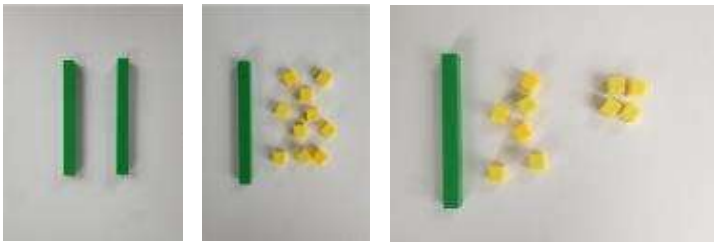
Adding 1, 2, 3 more *Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as;* *'1 more than 5 is equal to 6.'* *'2 more than 5 is equal to 7.'* *'8 is 3 more than 5.'* ***Over time, pupils should be encouraged to rely more on their number bonds knowledge than on counting strategies.***	1 more than 5 5 + 1 = 6  2 more than 5 5 + 2 = 7  
Adding three single digit numbers (make ten first) *Pupils may need to try different combinations before they find the two numbers that make 10.* *The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.* *The second bead string shows 4, 6 and then 7.* *The final bead string shows how they have now been put together to find the total.*	 

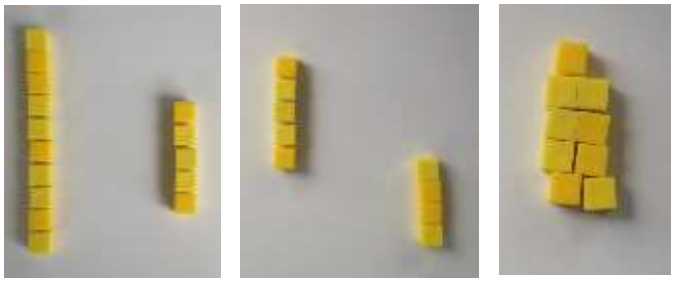
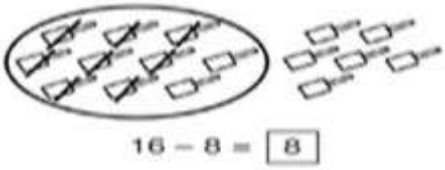
Partitioning to add (no regrouping) *Place value grids and Dienes blocks could be used as shown in the diagram before moving onto pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.* *When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column addition with formal recording.*	24 + 13 = 37 
Introducing column method for addition, regrouping only *Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.* *See additional guidance on unit pages for extra guidance on this strategy.*	24 + 17 

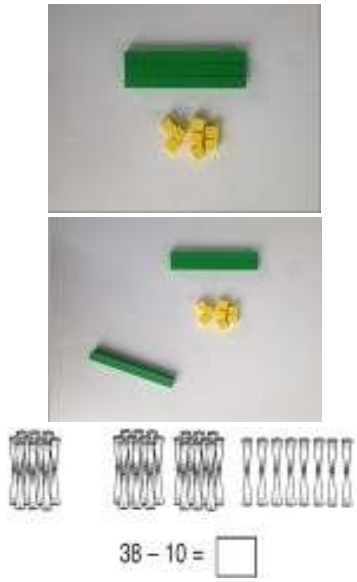
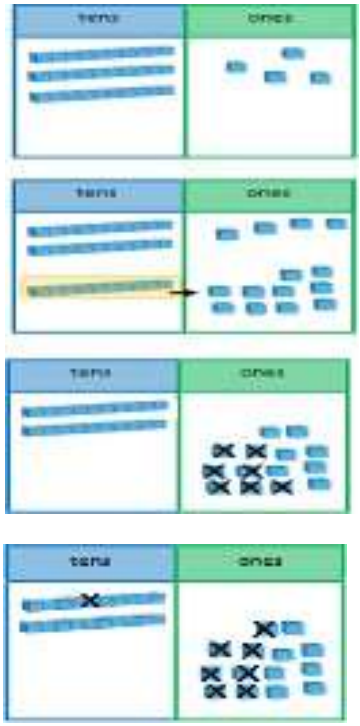
Adding multiples of ten *Using the vocabulary of 1 ten, 2 tens, 3 tens etc.* ***<u>alongside</u>*** *10, 20, 30 is important, as pupils need to understand that it is a* ***ten*** *and not a one that is being added and they need to understand that a '2' digit in the tens column has a value of twenty.* *It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.*	50 = 30 + 20   3 tens + 5 tens = _____ tens 30 + 50 = _____  36 + 40 = ☐

Y1 Subtraction

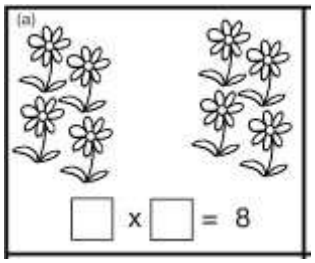
Strategy & guidance	CPA
Taking away from the ones *When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes.*	   
Counting back *Subtracting 1, 2, or 3 by counting back* ***Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.***	  16 – 2 = 14  

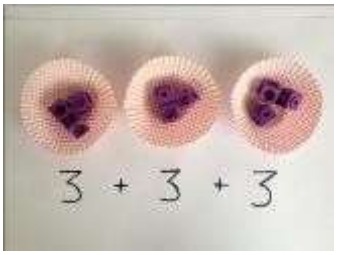
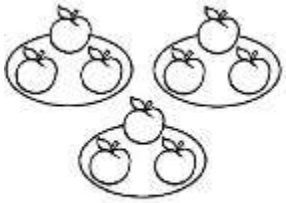
Part-part-whole *Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils start with ten cubes placed on the whole.* *They then remove what is being taken away from the whole and place it on one of the parts.* *The remaining cubes are the other part and also the answer. These can be moved into the second part space.*	10 − 6 = 4  
Make ten strategy *To subtract a 1-digit number from a 2-digit number.* *Pupils identify how many need to be taken away to make ten first, partitioning the number being subtracted. Then they take away the rest to reach the answer.*	14 – 5 = 9  
Regroup a ten into 10 ones *After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will support pupils when they later use the column method.*	  20 − 4 =

Taking away from the tens *Pupils should identify that they can also take away from the tens and get the same answer.* *This reinforces their knowledge of number bonds to 10 and develops their application of number bonds for mental strategies.*	9 = 15− 6   16 − 8 = 8
Partitioning to subtract without regrouping *Dienes blocks on a place value chart (developing into using images on the chart) could be used, as when adding 2-digit numbers, reinforcing the main concept of place value for Year 1.* *When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column subtraction with formal recording.*	34 − 13 = 21  34 – 13 = 21

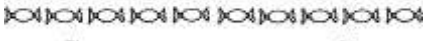
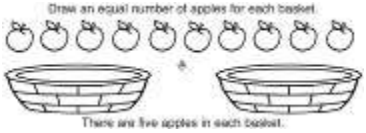
Subtracting multiples of ten *Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a* ***ten*** *not a one that is being taken away.*	40 = 60 – 20  	38 − 10 = 28 
Column method with regrouping *This example shows how pupils should work practically when being introduced to this method.* *There is no formal recording in columns in Year 1 but this practical work will prepare pupils for formal methods in Year 2.* *See additional guidance on unit pages to support with this method.*	34 − 17 = 17  	

Y1 Multiplication

Strategy & guidance	CPA
Skip counting in multiples of 2, 5, 10 from zero *The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10.* *Count the groups as pupils are skip counting.* *Number lines can be used in the same way as the bead string.* *Pupils can use their fingers as they are skip counting.*	 4 × 5 = 20  2 × 4 = 8
Making equal groups and counting the total *How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2. The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.*	  Draw  to show 2 x 3 = 6

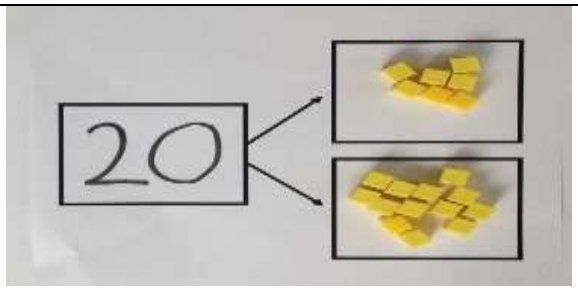
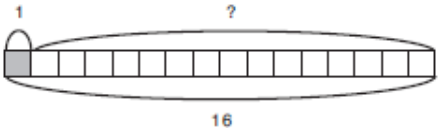
Solve multiplications using repeated addition *This strategy helps pupils make a clear link between multiplication and division as well as exemplifying the 'repeated addition' structure for multiplication. It is a natural progression from the previous 'count all' strategy as pupils can be encouraged to 'count on'. However, as number bonds knowledge grows, pupils should rely more on these important facts to calculate efficiently.*	3 x 3 = 3 + 3 + 3   How many apples are there altogether? 3 + 3 + 3 = 9

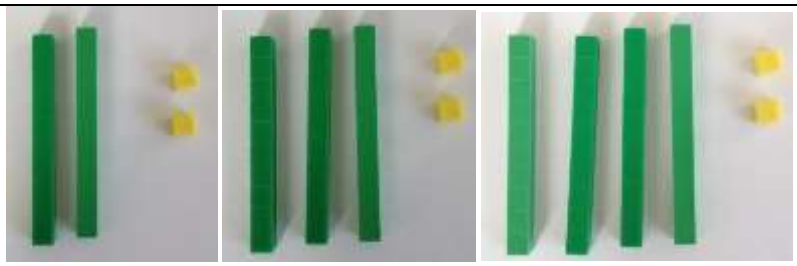
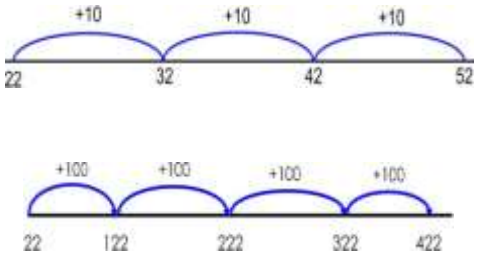
Y1 Division

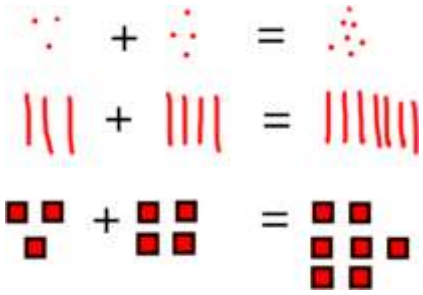
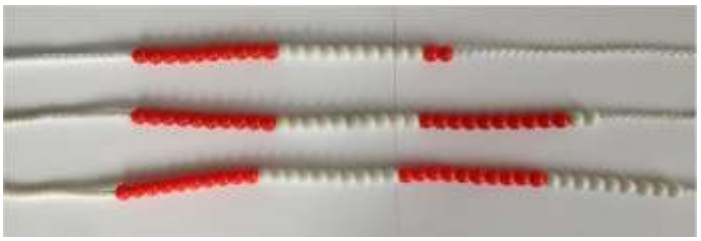
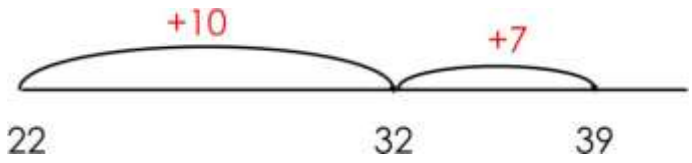
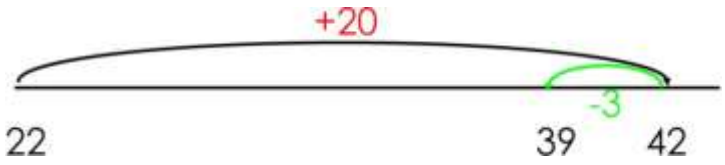
Strategy & guidance	CPA
Sharing objects into groups *Pupils should become familiar with division equations through working practically.* *The division symbol is not formally taught at this stage.*	10 ÷ 2 = 5   There are 10 sweets. Ring groups of 2.  There are _____ groups of 2. Draw an equal number of apples for each basket.  There are five apples in each basket.

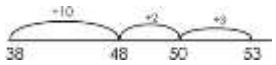
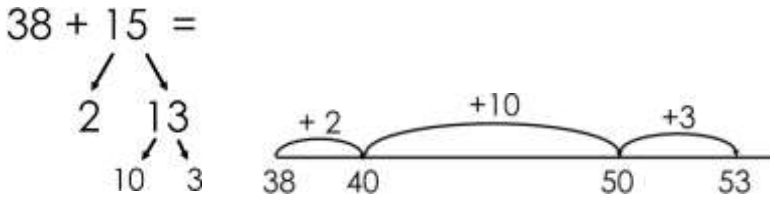
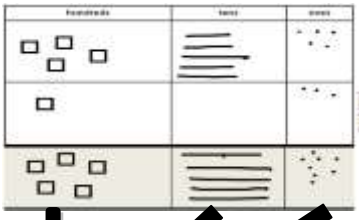
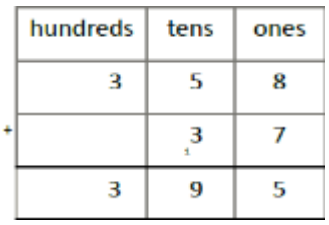
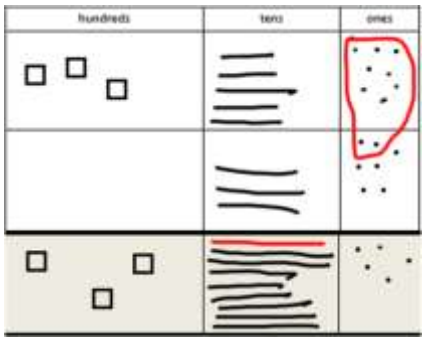
Progression in calculations Year 2

Y2 Addition

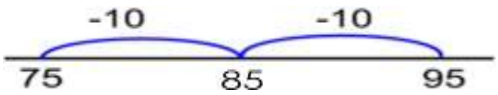
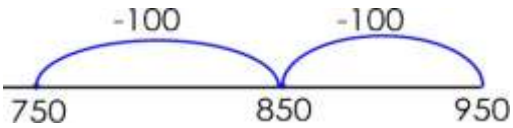
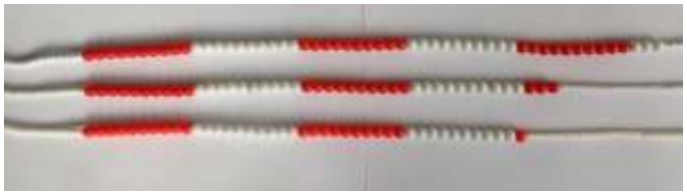
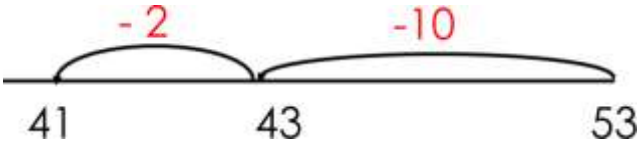
Strategy & guidance	CPA
Part-part-whole *Pupils explore the different ways of making 20. They can do this with all numbers using the same representations.* *This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.*	 20 = 17 + 3 20 = 3 + 17 20 – 3 = 17 20 – 17 = 3   □ + □ = 20 20 – □ = □ □ + □ = 20 20 – □ = □ □ + 1 = 16 16 – 1 = □ 1 + □ = 16 16 – □ = 1 

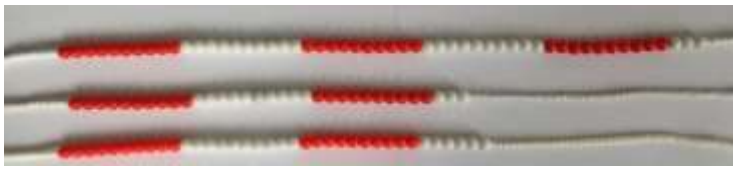
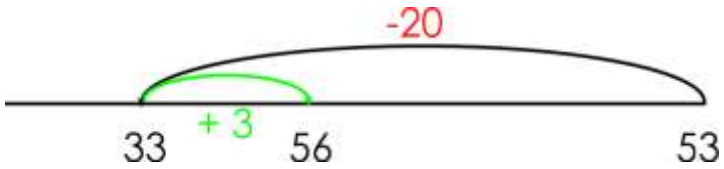
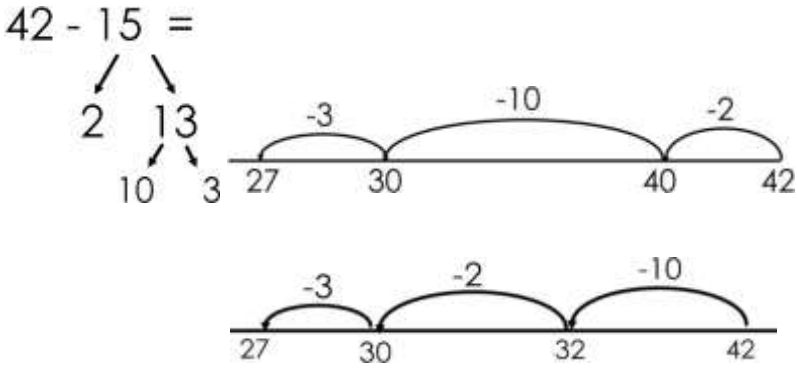
Counting on in tens and hundreds	 

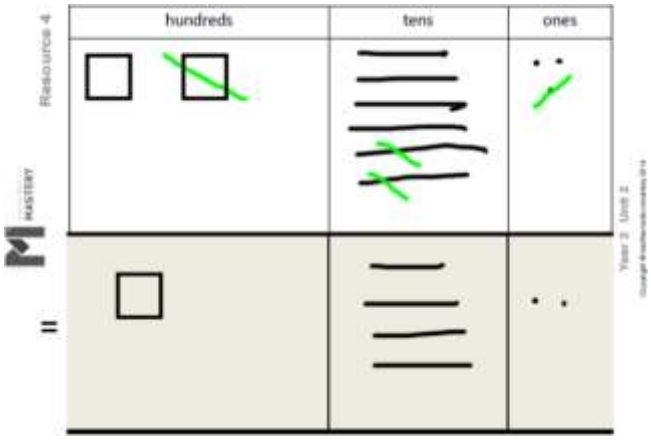
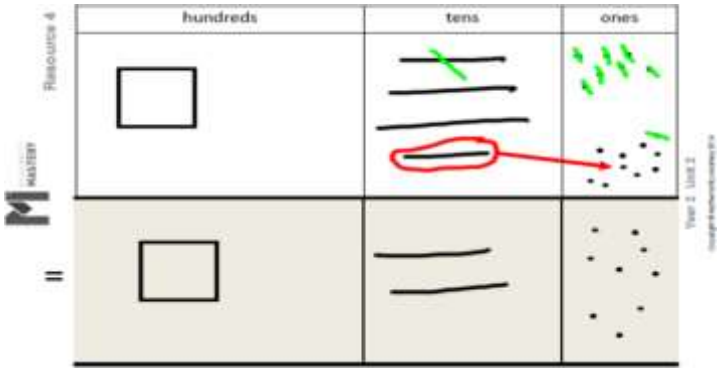
Strategy & guidance	CPA
Using known facts to create derived facts *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.*	 3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Partitioning one number, then adding tens and ones *Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.*	  22 + 17 = 39
Round and adjust (sometimes known as a compensating strategy) *Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.*	  22 + 17 = 39

Strategy & guidance	CPA
Make ten strategy  *How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.* *It relies on an understanding that numbers can be partitioned in different ways in order to easily make a multiple of ten.*	 
Partitioning to add without regrouping *As in Year 1, this is a mental strategy rather than a formal written method. Pupils use the Dienes blocks (and later, images) to represent 3-digit numbers but do not record a formal written method if there is no regrouping.*	 4 5 5 + 1 0 3 = 5 5 8
Column method with regrouping *Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations.* *As in Year 1, the focus for the column method is to develop a strong understanding of place value.*	 

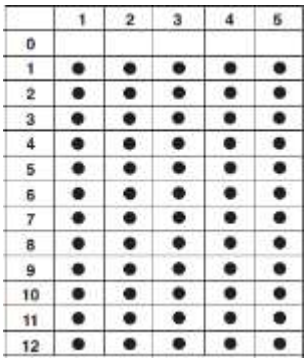
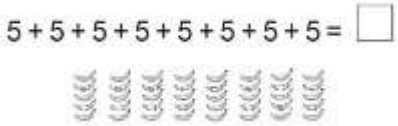
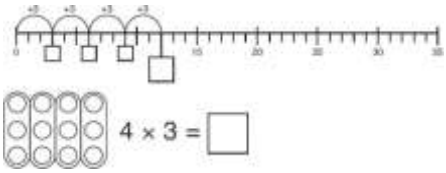
Y2 Subtraction

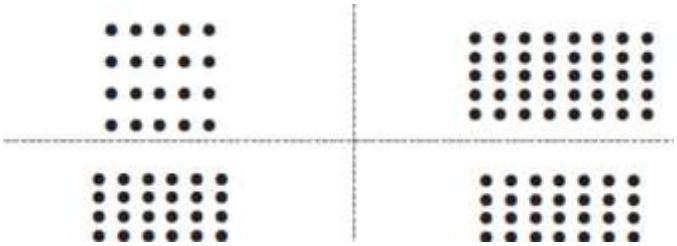
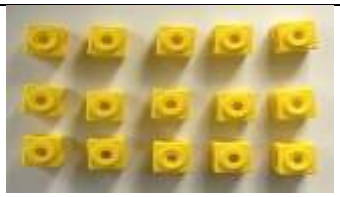
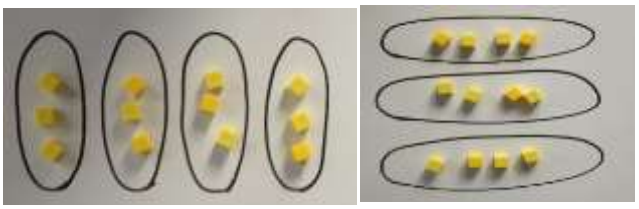
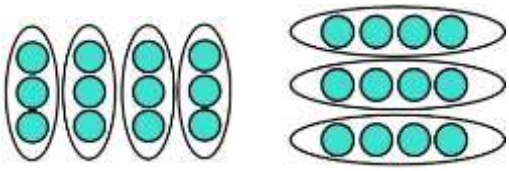
Strategy & guidance	CPA
Counting back in multiples of ten and one hundred	  
Using known number facts to create derived facts *Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy, encouraging pupils to apply their knowledge of number bonds to add multiples of ten and 100.*	 8 − 4 = 4 *leads to* 80 − 40 = 40 *leads to* 800 − 400 = 400
Subtracting tens and ones *Pupils must be taught to partition the second number for this strategy as partitioning both numbers can lead to errors if regrouping is required.*	53 − 12 = 41  

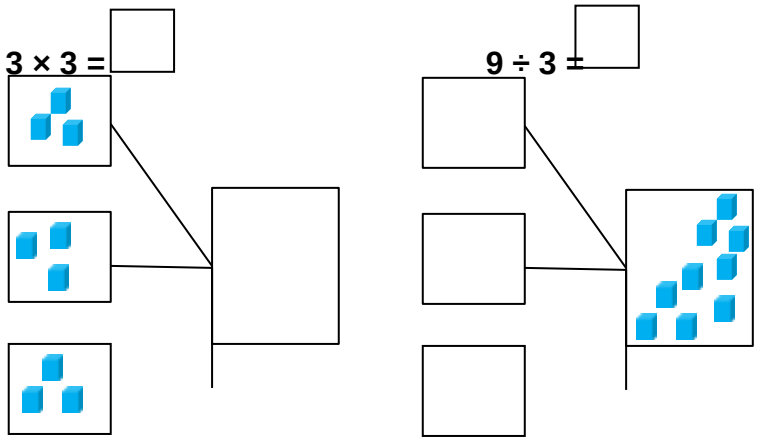
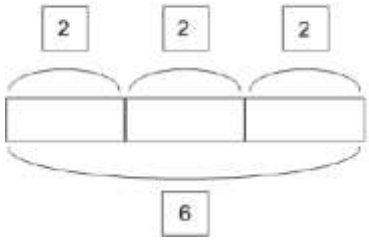
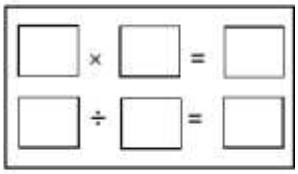
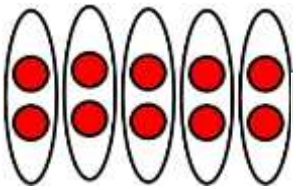
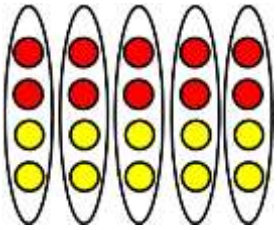
Strategy & guidance	CPA
Round and adjust (sometimes known as a compensating strategy) *Pupils must be taught to round the number that is being subtracted.* *Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.*	  53 − 17 = 36
Make ten *How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.* *It relies on an understanding that numbers can be partitioned in different ways in order to subtract to a multiple of ten.* *Pupils should develop an understanding that the parts can be added in any order.*	 

Strategy & guidance	CPA
Partitioning to subtract without regrouping *As in Year 1, the focus is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial.* *Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.*	 263 – 121= 142
Column method with regrouping *The focus for the column method is to develop a strong understanding of place value and concrete manipulatives should be used alongside.* *Pupils are introduced to calculations that require two instances of regrouping (initially from tens to one and then from hundreds to tens). E.g. 232 – 157 and are given plenty of practice using concrete manipulatives and images alongside their formal written methods, ensuring that important steps are not missed in the recording.* *Caution should be exercised when introducing calculations requiring 'regrouping to regroup' (e.g. 204 – 137) ensuring ample teacher modelling using concrete manipulatives and images.*	hundreds tens ones 1 ³~~4~~ ¹7 - 1 8 1 2 9 

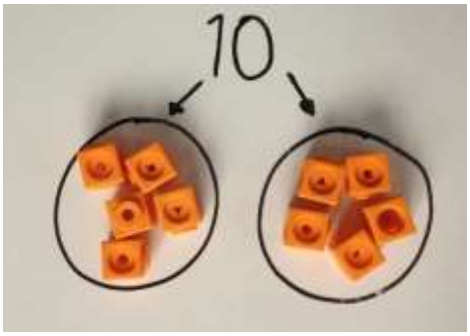
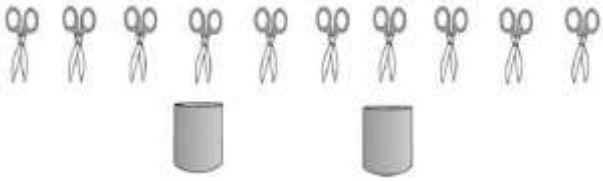
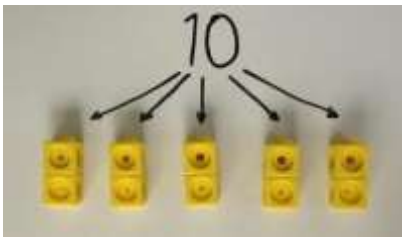
Y2 Multiplication

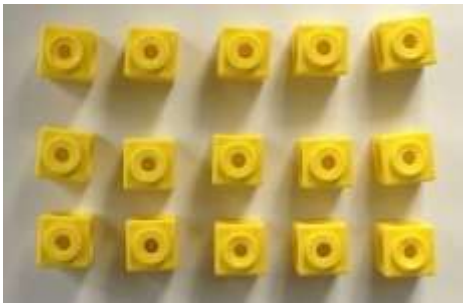
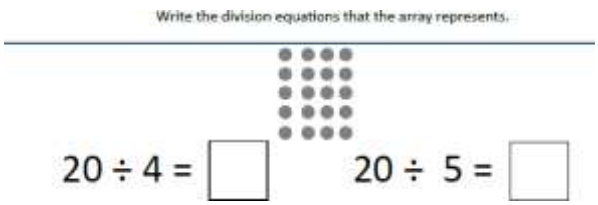
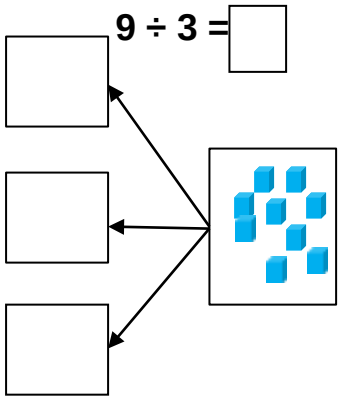
Strategy & guidance	CPA
Skip counting in multiples of 2, 3, 4, 5, 10 from zero *Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'.* *Dot arrays can be used to create a visual representation for the different multiplication facts. Bead strings, groups of cubes (or unifix / multilink towers) provide useful concrete representations.*	 
Multiplication as repeated addition *Pupils apply skip counting to help find the totals of repeated additions.*	   

Strategy & guidance	CPA
Arrays to represent multiplication equations *Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful.* *Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.* *The relationship between multiplication and division also begins to be demonstrated.*	  
Multiplication is commutative *Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.*	    12 = 3 × 4 12 = 4 × 3

Strategy & guidance	CPA
Use of part-part-whole model to establish the inverse relationship between multiplication and division *This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the value of the parts and the number of parts.* *It is important to highlight that with multiplication, the parts are of equal value as this is different to how this model is used for addition and subtraction.*	There are three equal parts. Each part has a value of three. What is the whole?  What multiplication and division equations can you write for each bar model? Prove that the equations are correct using a bead string.  
Doubling to derive new multiplication facts *Pupils learn that known facts from easier multiplication tables can be used to derive facts from related times tables using doubling as a strategy.* *At this stage they double the 2× table facts to derive the 4× table facts.*	 5 × 2 = 10  5 × 4 = 20

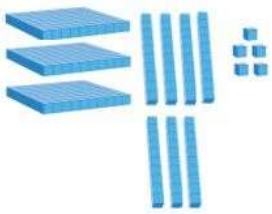
Y2 Division

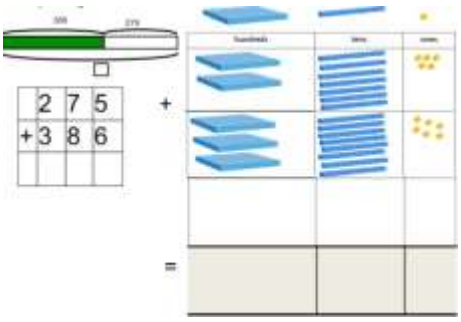
Strategy & guidance	CPA
Division as sharing *Here, division is shown as sharing.* *If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.*	10 ÷ 2 = 5  
Division as grouping *Here, division is shown as grouping.* *If we have ten forks and we put them into groups of two, there are 5 groups.*	10 ÷ 2 = 5  

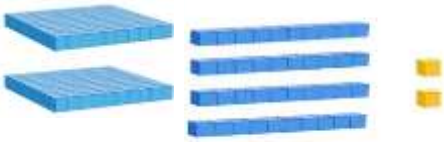
Strategy & guidance	CPA
Use of part-part-whole model to represent division equations and to emphasise the relationship between division and multiplication *Pupils use arrays of concrete manipulatives and images of familiar objects to solve division equations.* *They begin to use dot arrays to develop a more abstract concept of division.* *It is important to highlight that with multiplication and division, the parts are of equal value as this is different to how this model is used for addition and subtraction.*	  15 ÷ 5 = 3 15 ÷ 3 = 5  Write the division equations that the array represents. 20 ÷ 4 = ☐ 20 ÷ 5 = ☐ The whole is nine. There are three equal parts. What is the value of each part? **9 ÷ 3 =** ☐ 

Progression in calculations Year 3

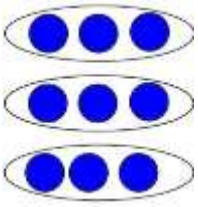
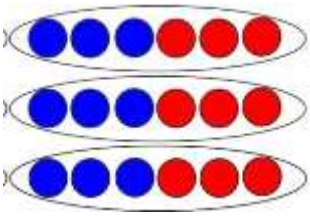
Y3 Addition & Subtraction

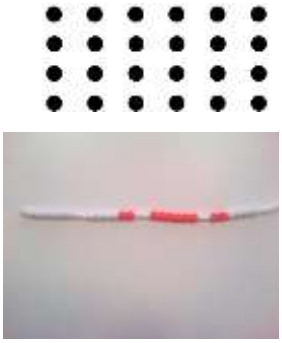
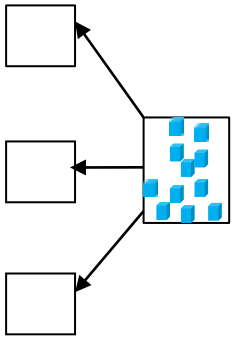
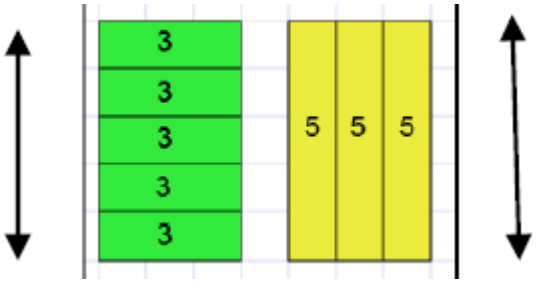
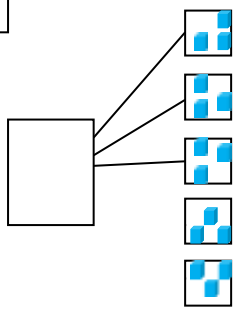
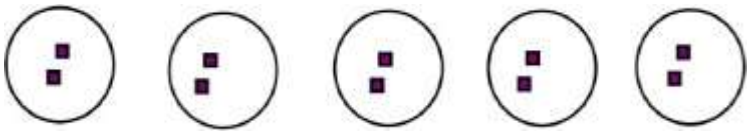
Strategy & guidance	CPA
Add and subtract numbers mentally, including: • **a three-digit number and ones;** • **a three-digit number and tens;** • **a three-digit number and hundreds** *Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.* *To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid written algorithms, including (but not exhaustively):* • *known number facts within 20,* • *derived number facts,* • *'Make ten',* • *round and adjust* *See Year 2 guidance for exemplification of these – the use of concrete manipulatives other than Dienes blocks is important in reinforcing the use of these strategies.* *It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.*	It is important to model the mental strategy using concrete manipulatives in the first instance and pupils should be able to exemplify their own strategies using manipulatives if required, with numbers appropriate to the unit they are working on (3-digit numbers in Units 1 & 4; 4-digit numbers in Unit 13). However, pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulatives or images. <u>No regrouping</u> 345 + 30 274 - 50 1128 + 300 1312 - 300 326 + 342 856 - 724  I know 4 + 3 = 7, so 4 tens plus 3 tens is equal to 7 tens. 345 + 30 = 375. <u>With some regrouping</u> 416 + 25 232 - 5 383 + 130 455 - 216 611 + 194 130 - 40 1482 + 900 2382 - 500

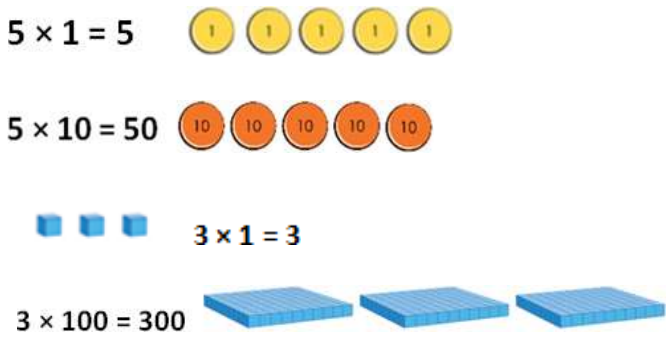
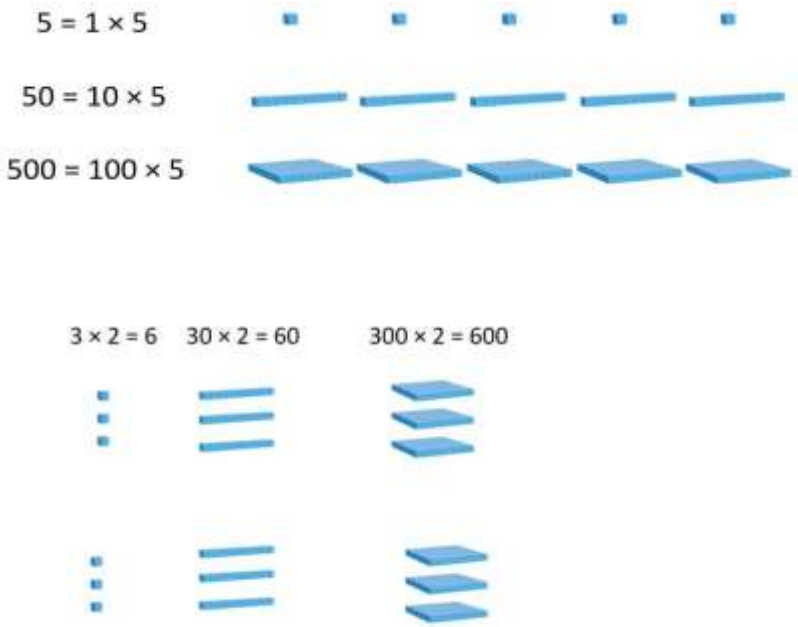
Strategy & guidance	CPA
Written column method for calculations that require regrouping with up to 4-digits *Dienes blocks should be used alongside the pictorial representations during direct teaching and can be used by pupils both for support and challenge. Place value counters can also be introduced at this stage.* *This work revises and reinforces ideas from Key Stage 1, including the focus on place value – see Year 2 exemplification.* *Direct teaching of the columnar method should require at least one element of regrouping, so that pupils are clear about when it is most useful to use it. Asking them 'Can you think of a more efficient method?' will challenge them to apply their number sense / number facts to use efficient mental methods where possible.* *As in Year 2, pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping. In Year 3 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should be challenged as to whether this is the most efficient method, considering whether mental methods (such as counting on, using known number facts, round and adjust etc.) may be likelier to produce an accurate solution.* *Pupils requiring support might develop their confidence in the written method using numbers that require no regrouping.* *See Unit materials for extra guidance on this strategy.*	As for the mental strategies, pupils should be exposed to concrete manipulatives modelling the written calculations and should be able to represent their written work pictorially or with concrete manipulatives when required. Again, they should be encouraged to calculate with known and derived facts and should not rely on counting images or manipulatives.  5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one. <u>Regrouping (including multiple separate instances)</u> 672 + 136 734 – 82 468 + 67 831 - 76 275 + 386 435 – 188 <u>'Regrouping to regroup'</u> 204 – 137 1035 - 851

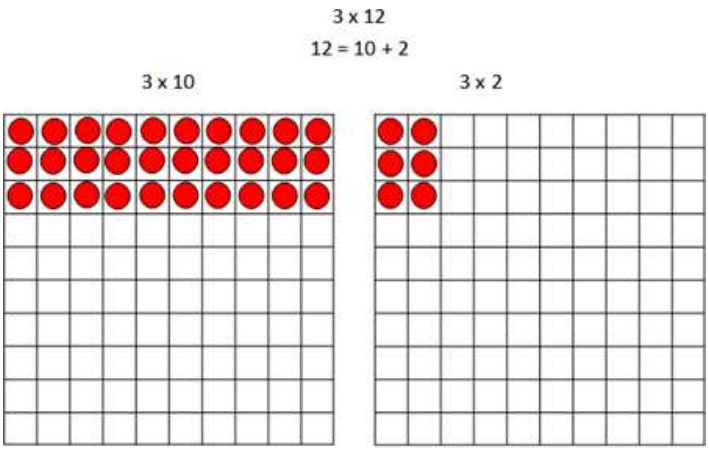
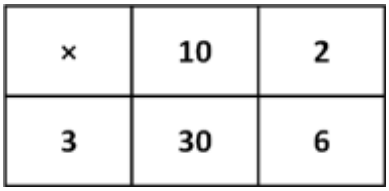
Strategy & guidance	CPA
Find 10, 100 more or less than a given number *As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (and, in the last unit of the Summer term, thousands), including use of concrete manipulatives and appropriate images.* *After initial teaching, this should be incorporated into transition activities and practised regularly.*	142 + 100 = 242  

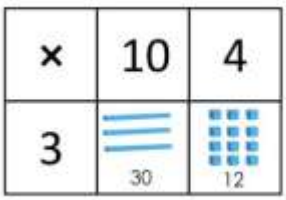
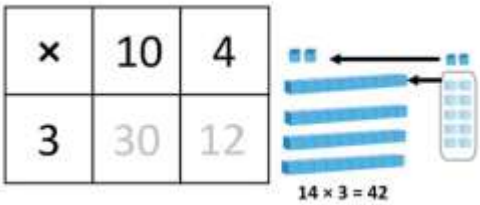
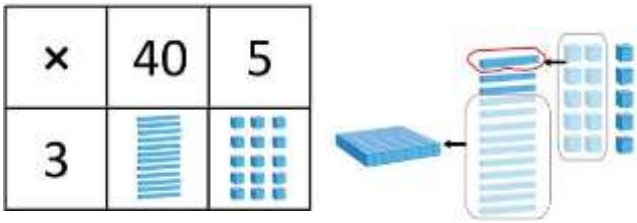
Y3 Multiplication

Strategy & guidance	CPA
Doubling to derive new multiplication facts *Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.* *This builds on the doubling strategy from Year 2.*	3 x 3 = 9  3 x 6 = double 9 = 18 

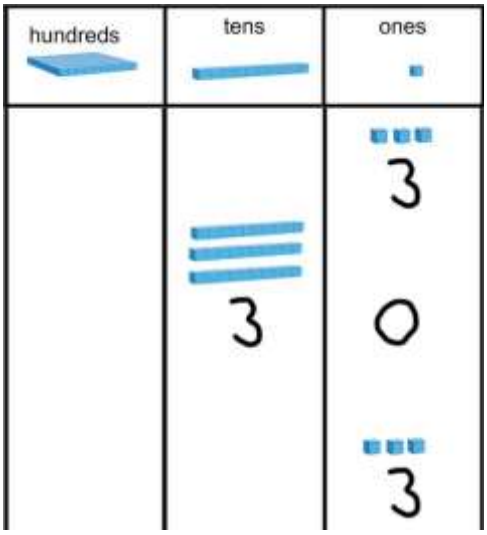
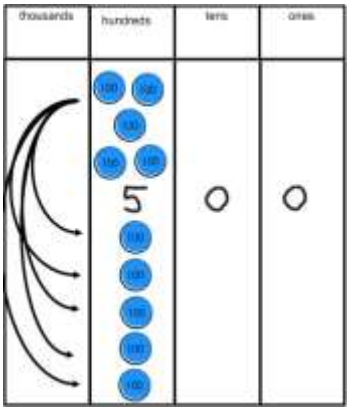
Strategy & guidance	CPA
Skip counting in multiples of 2, 3, 4, 5, 6, 8 and 10 *Rehearsal of previously learnt tables as well as new content for Year 3 should be incorporated into transition activities and practised regularly.*	
Use of part-part-whole model with arrays and bar models to establish commutativity and inverse relationship between multiplication and division *In these contexts pupils are able to identify all the equations in a fact family.*	   
Ten times greater *Pupils's work on this must be firmly based on concrete representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.*	 

Strategy & guidance	CPA
Multiplying by 10 and 100 *Building on the ten times greater work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.*	
Using known facts for multiplying by multiples of 10 and 100 *Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.* *It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)*	

Strategy & guidance	CPA
Multiplication of 2-digit numbers with partitioning (no regrouping) *Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.* *Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.*	 Now add the total number of tens and ones   **3 x 12 = 36**

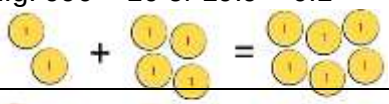
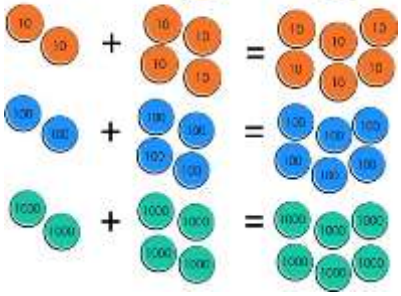
Strategy & guidance	CPA
Multiplication of 2-digit numbers with partitioning (regrouping) *Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written methods in Year 4.* *Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.* *Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.*	  

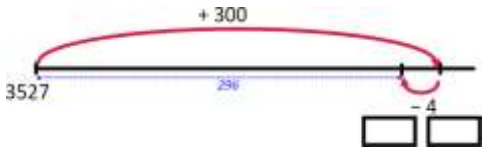
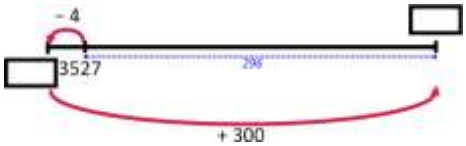
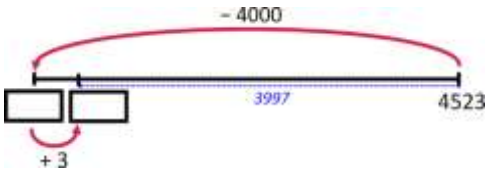
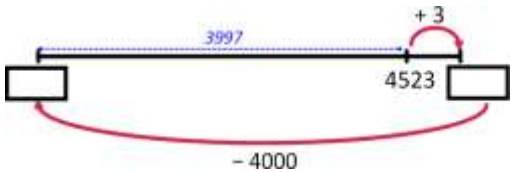
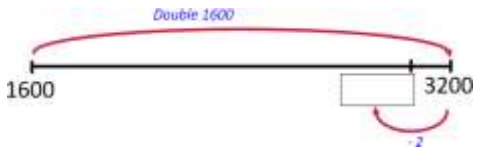
Y3 Division

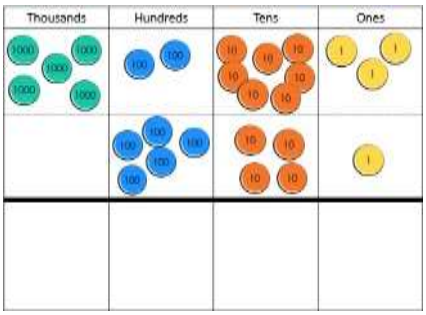
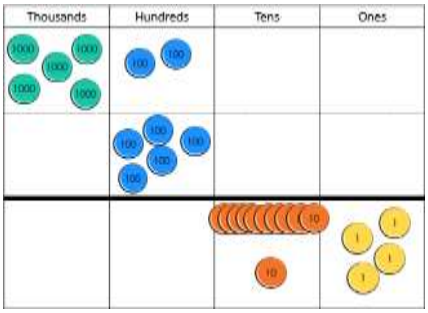
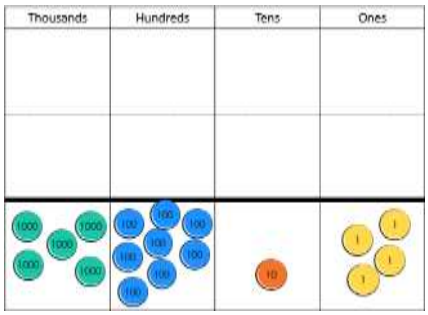
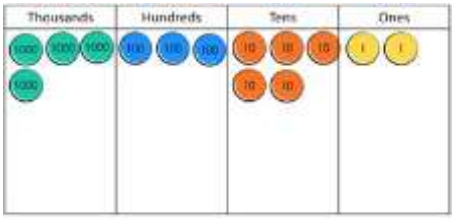
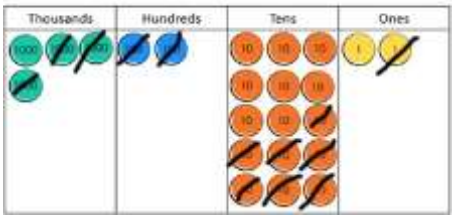
Strategy & Guidance	CPA
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down *Pupils use the strategy of 'scaling down', representing numbers with concrete manipulatives and making the value ten times smaller.*	 **3 × 10 = 30** **30 ÷ 10 = 3**
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping *Pupils divide by 10, 100 and 1000 by making groups of the divisor.*	500 ÷ 100 = ☐ My whole is 500 and the value of the equal parts is 100. How many parts are there? 

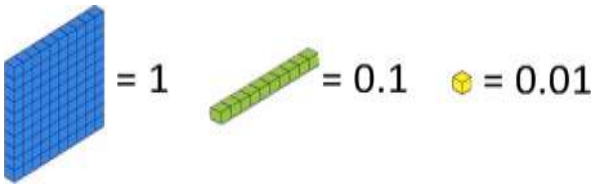
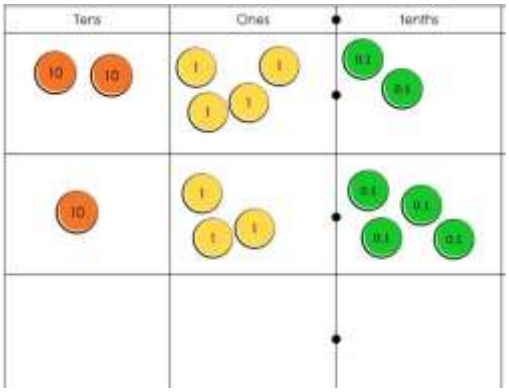
Progression in calculations Year 1

Y4 Addition & Subtraction

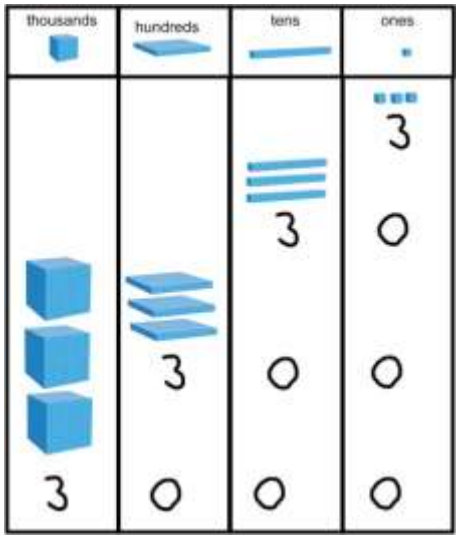
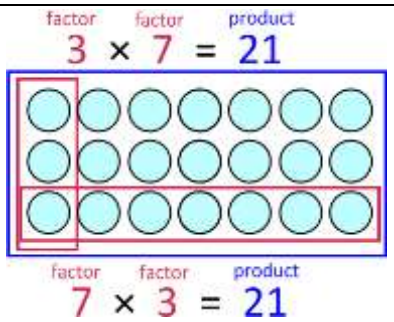
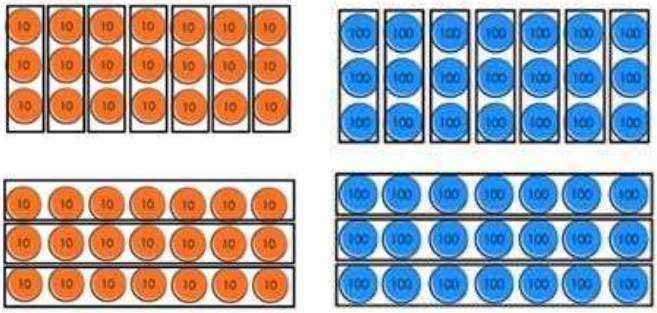
Strategies & Guidance	CPA
Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000. *Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.* **Count forwards and backwards in tenths and hundredths**	  Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. E.g. 990 + 10 or 19.9 + 0.1 
Using known facts and knowledge of place value to derive facts. **Add and subtract multiples of 10, 100 and 1000 mentally** *Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.*	 2 + 4 = 6 20 + 40 = 60 200 + 400 = 600 2000 + 4000 = 6000
Adding and subtracting by partitioning one number and applying known facts. *By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.*	See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery.

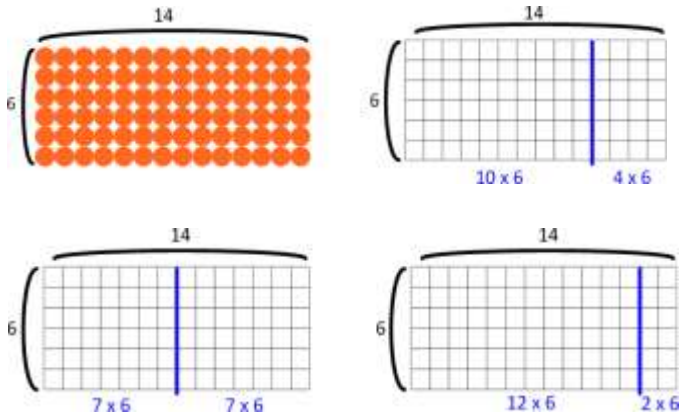
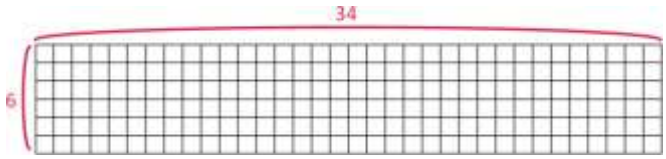
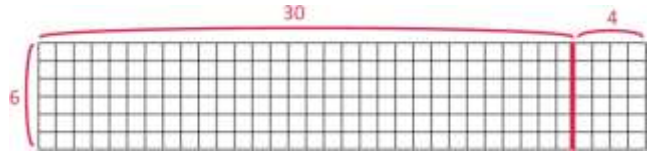
Strategies & Guidance	CPA
Round and adjust *Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.* *It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.* *Build flexibility by completing the same calculation in a different order.*	3527 + 296 = 3827 - 4  Completing the same calculation but adjusting first: 3527 + 296 = 3523 + 300  4523 – 3997 = 523 + 3  Completing the same calculation but adjusting first: 4523 – 3997 = 4526 - 4000 
Near doubles *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.*	1600 + 1598 = double 1600 – 2 

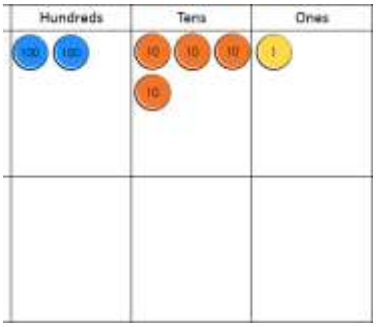
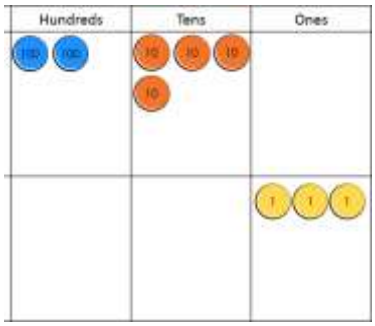
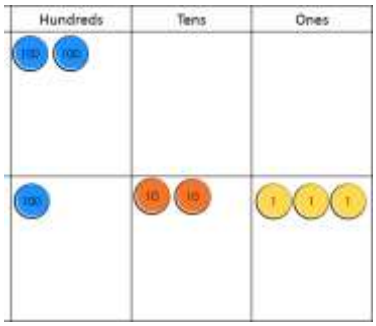
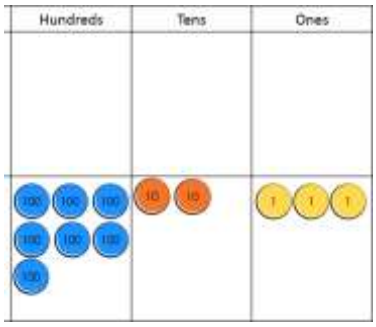
Strategies & Guidance	CPA
Written column methods for addition *Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.*	   5 2 7 3 + 5 4 1 1 5 8 1 4
Written column methods for subtraction *Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.*	  4 2 3 1 5 2 - 3 2 7 1 1 0 8 1

Strategies & Guidance	CPA
Calculating with decimal numbers *Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.* *Place value counters are another useful manipulative for representing decimal numbers.* *All of the calculation strategies for integers (whole numbers) can be used to calculate with decimal numbers.*	 24.2 + 13.4 = 

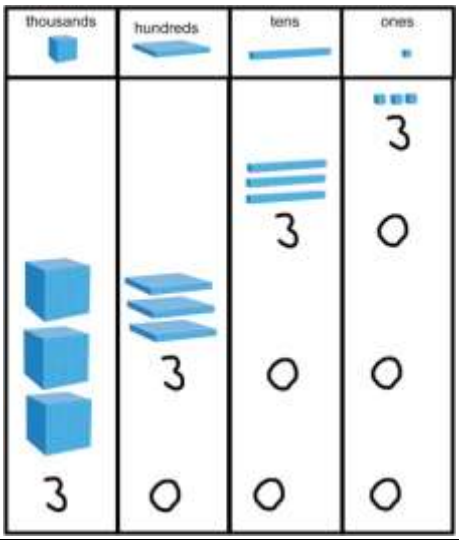
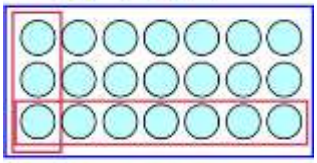
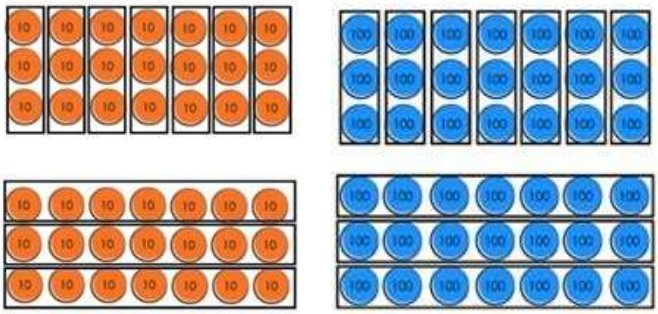
Y4 Multiplication

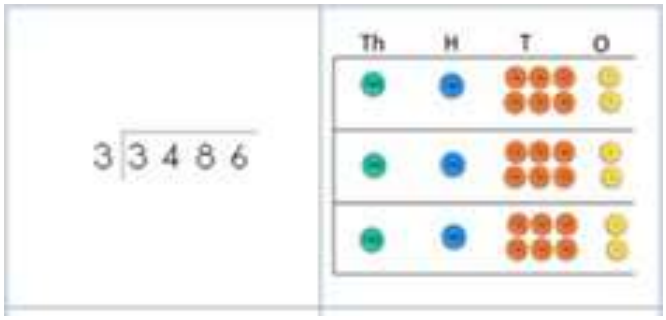
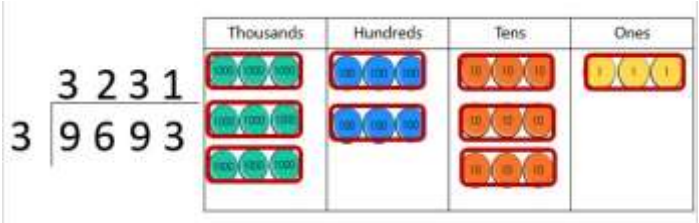
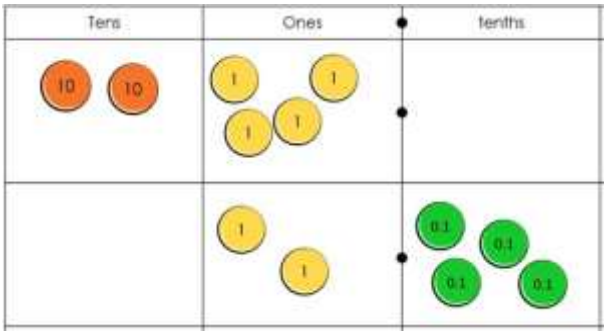
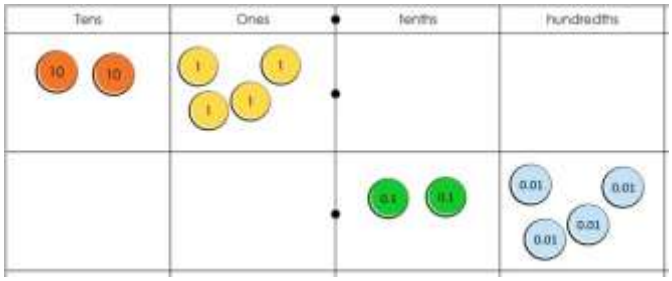
Strategies & Guidance	CPA
Multiplying by 10 and 100 *When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.* *When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.* *Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000*	 3 x 10 = 30 3 x 100 = 300 3 x 1000 = 3000
Using known facts and place value for mental multiplication involving multiples of 10 and 100 *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.*	  30 x 7 = 210 300 x 7 = 2100 70 x 3 = 210 700 x 3 = 2100 7 x 30 = 210 7 x 300 = 2100 3 x 70 = 210 3 x 700 = 2100

Strategies & Guidance	CPA
Multiplying by partitioning one number and multiplying each part *Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations.* *As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.* **Distributive law** **a x (b + c) = a x b + a x c**	14 x 6  34 × 6  30 × 6 + 4 × 6 
Mental multiplication of three 1-digit numbers, using the associative law *Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.*	Four pots each containing two flowers which each have seven petals. How many petals in total?  (4 x 2) x 7 or 4 x (2 x 7)

Strategies & Guidance	CPA
Short multiplication of 3-digit number by 1-digit number *To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.* *With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.*	Exemplification of this process is best understood through viewing the video tutorial  To calculate 241 x 3, represent the number 241. Multiply each part by 3, regrouping as needed.  2 4 1 x 3 7 2 3 1  

Y4 Division

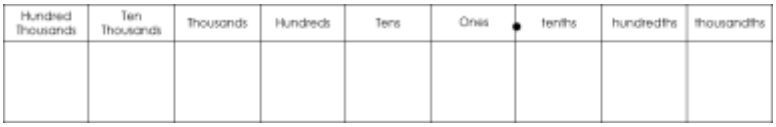
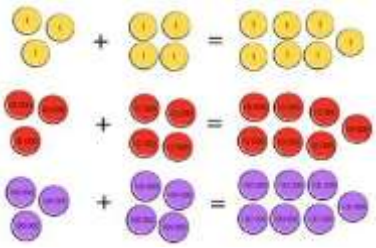
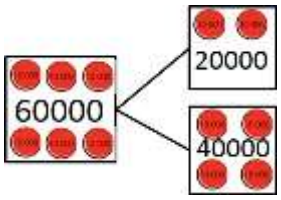
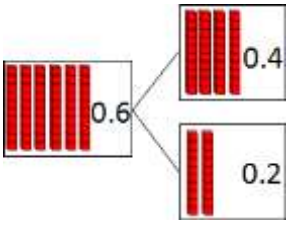
Strategies & Guidance	CPA
Dividing by 10 and 100 *When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.* *When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21*	 30 ÷ 10 = 3 300 ÷ 100 = 3 3000 ÷ 1000 = 3 300 ÷ 10 = 30 3000 ÷ 100 = 30 3000 ÷ 10 = 300
Derived facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	21 ÷ 3 = 7  21 ÷ 7 = 3  210 ÷ 7 = 30 210 ÷ 3 = 70 210 ÷ 30 = 7 210 ÷ 70 = 3 2100 ÷ 7 = 300 2100 ÷ 3 = 700 2100 ÷ 300 = 7 2100 ÷ 700 = 3

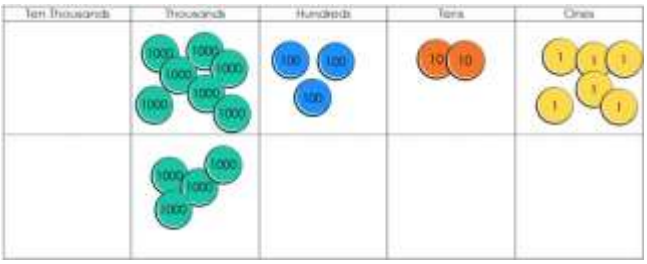
Strategies & Guidance	CPA
Short division of 4-digit numbers by 1-digit numbers *Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding.* *They progress to calculations that require regrouping in the hundreds or tens columns.* *Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.*	**Division as sharing**  **Division as grouping** 
Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths *When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.*	24 ÷ 10 = 2.4  24 ÷ 100 = 0.24 

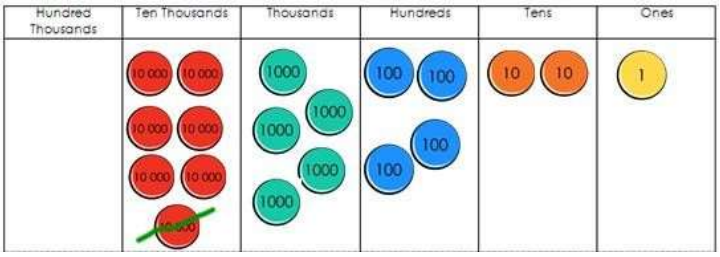
Progression in calculations Year 5 + Year 6

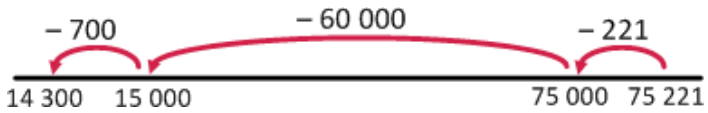
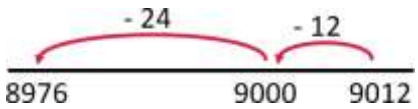
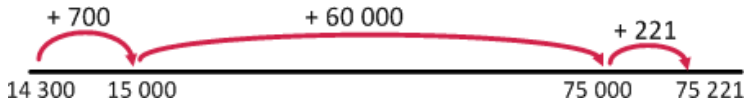
Year 5 and Year 6 are together because the calculation strategies used are broadly similar, with Year 6 using larger and smaller numbers. Any differences for Year 6 are highlighted in red.

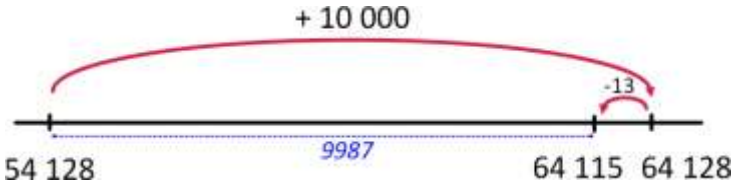
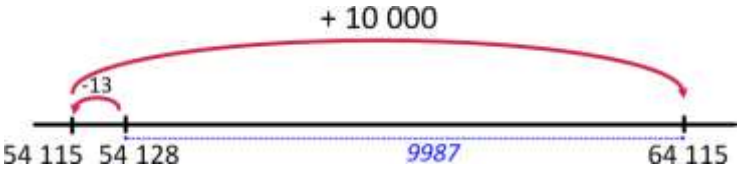
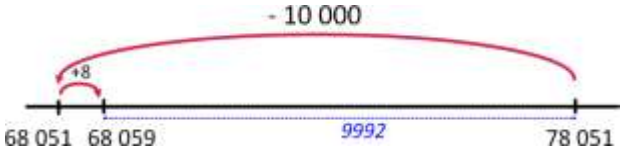
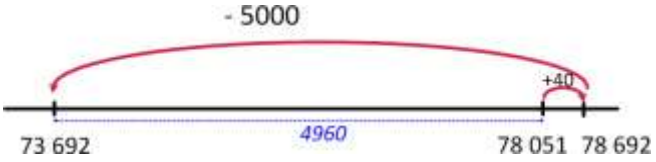
Y5 and Y6 Addition & Subtraction

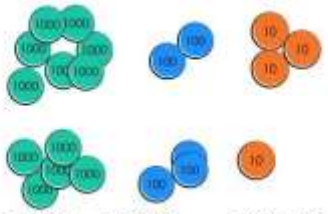
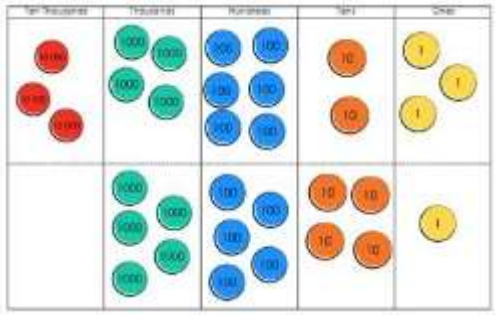
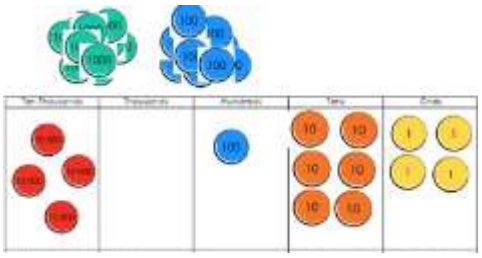
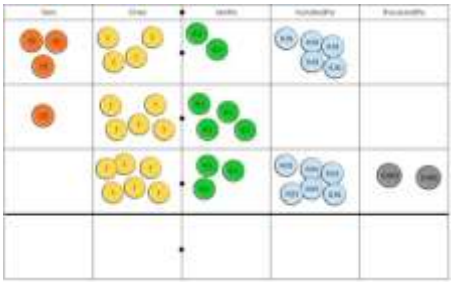
Strategies & Guidance	CPA
Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 *Skip counting forwards and backwards in steps of powers of 10 (i.e. 10, 100, 1000, 10 000 and 100 000) should be incorporated into transition activities and practised regularly.* *In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.* *In Year 6 pupils work with numbers up to 10 000 000.*	Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed.  Counting sticks and number lines:   Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. e.g. 9900 + 100 = 10 000 or 99 000 + 1000 = 100 000
Using known facts and understanding of place value to derive *Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.* *In Year 5 extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.* *In Year 6 extend to multiples of one million.* *These derived facts should be used to estimate and check answers to calculations.*	 3 + 4 = 7 30 000 + 40 000 = 70 000 300 000 + 400 000 = 700 000  20 000 + 40 000 = 60 000 40 000 + 20 000 = 60 000 60 000 - 40 000 = 20 000 60 000 - 20 000 = 40 000  0.6 = 0.2 + 0.4 0.6 = 0.4 + 0.2 0.2 = 0.6 - 0.4 0.4 = 0.6 - 0.2

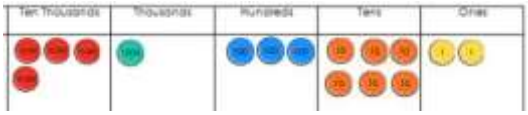
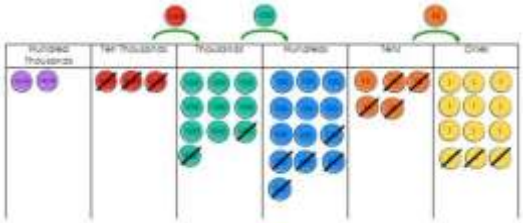
Strategies & Guidance	CPA
Partitioning one number and applying known facts to add. *Pupils can use this strategy mentally or with jottings as needed.* *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.* *They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.* *Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.*	**Partitioning into place value amounts (canonical partitioning):** 4650 + 7326 = 7326 + 4000 + 600 + 50  With place value counters, represent the larger number and then add each place value part of the other number. The image above shows the thousands being added. Represent pictorially with an empty numberline:  **Partitioning in different ways (non-canonical partitioning):** Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count on to a multiple of 10. 6785 + 2325 = 6785 + 15 + 200 + 2110  The strategy can be used with decimal numbers, Make one: 14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3 

Strategies & Guidance	CPA
Subtraction by partitioning and applying known facts. *Pupils can use this strategy mentally or with jottings as needed.* *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be subtracted.* *Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.*	**Partitioning into place value amounts (canonical partitioning):** 75 221 – 14 300 = 75 221 – 10 000 – 4000 – 300  Represent pictorially with a number line, starting on the right and having the arrows jump to the left:  Develop understanding that the parts can be subtracted in any order and the result will be the same:  **Partitioning in different ways (non-canonical partitioning):** Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10. 

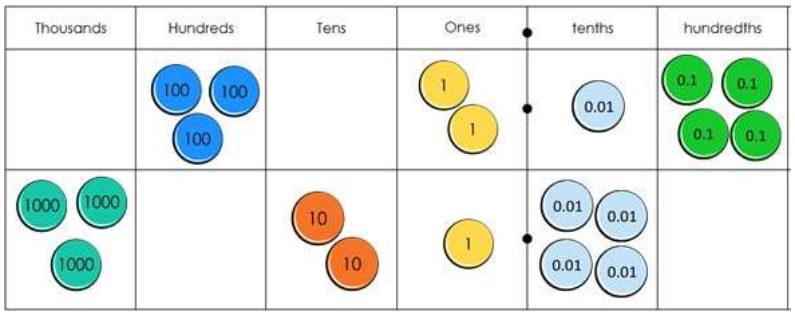
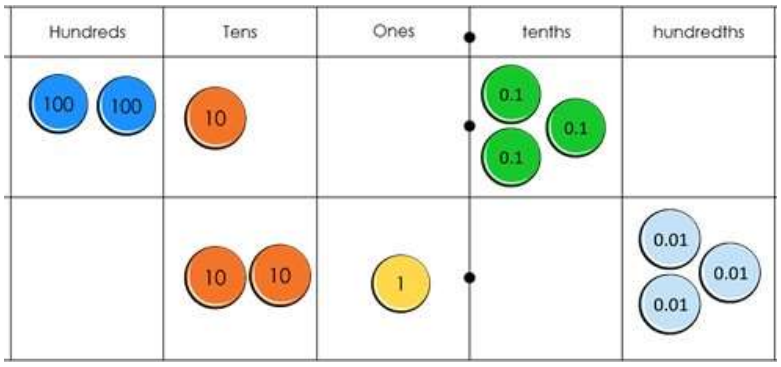
Strategies & Guidance	CPA
Calculate difference by "counting back" *It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts.*	75 221 – 14 300 Place the numbers either end of a numberline and work out the difference between them. Select efficient jumps.  Finding the difference is efficient when the numbers are close to each other: 9012 – 8976 
Calculate difference by "counting on" *Addition strategies can be used to find difference.*	75 221 – 14 300  Finding the difference is efficient when the numbers are close to each other 9012 – 8976 

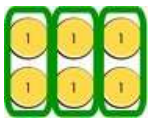
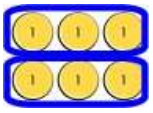
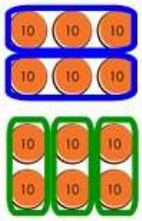
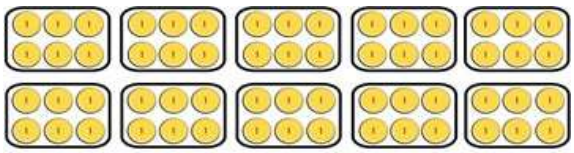
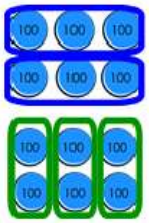
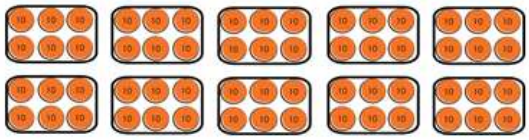
Strategies & Guidance	CPA
Round and adjust **Addition and subtraction using compensation** *Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding.* *It is very easy to be confused about how to adjust and so visual representations and logical reasoning are essential to success with this strategy.*	**Addition**  54 128 + 9987 = 54 128 + 10 000 – 13 = 64128 - 13 Pupils should realise that they can adjust first:  54128 + 9987 = 54128 – 13 + 10 000 = 54 115 + 10 000 **Subtraction**  78 051 – 9992 = 78 051 – 10 000 + 8 = 68 051 + 8 Pupils should realise that they can adjust first:  78 051 – 4960 = 78 051 + 40 – 5000 = 78 692 - 5000
Near doubles *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.*	160 + 170 = double 150 + 10 + 20 160 + 170 = double 160 + 10or 160 + 170 = double 170 - 10 2.5 + 2.6 = double 2.5 + 0.1

Strategies & Guidance	CPA
Partition both numbers and combine the parts *Pupils should be secure with this method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.* *If multiple regroupings are required, then pupils should consider using the column method.*	7230 + 5310 = 12 000 + 500 + 40 200 + 300 = 500  7000 + 5000 = 12000 30 + 10 = 40 Pupils should be aware that the parts can be added in any order.
Written column methods for addition *In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.* *Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.* *Continue to use concrete manipulatives alongside the formal method.* *When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.*	For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value. 3 4 6 2 3 + 5 5 4 1  Combine the counters in each column and regroup as needed: 3 4 6 2 3 + 5 5 4 1 1 1 4 0 1 6 4  Decimal numbers: 3 4 . 2 5 1 5 . 4 + 6 . 3 6 2 1 1 1 5 6 . 0 1 2 

Strategies & Guidance	CPA
Written column methods for subtraction *In Year 5, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.* *Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.* *In Year 3 and 4 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.*	4 1 3 6 2 - 3 2 2 4 3  3 1 5 1 4| 1 3 6| 2 - 3 2 2 4 3 9 1 1 9  The term regrouping should be the language used. You can use the terms 'exchange' with subtraction but it needs careful consideration. You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones). Or you can 'exchange' one of the tens for 10 ones resulting in 5 tens and 12 ones. If you have exchanged, then the number has been regrouped.

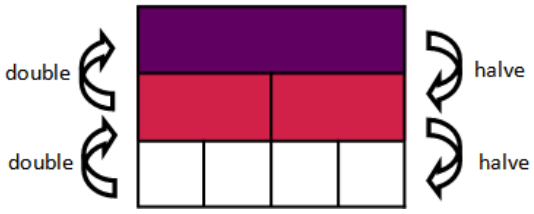
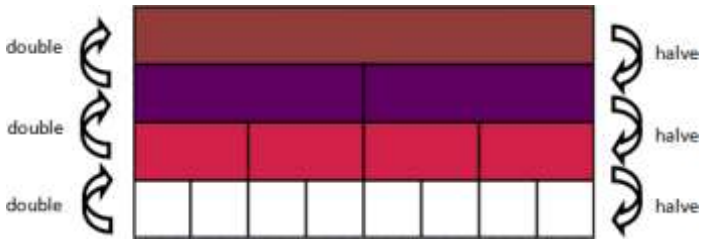
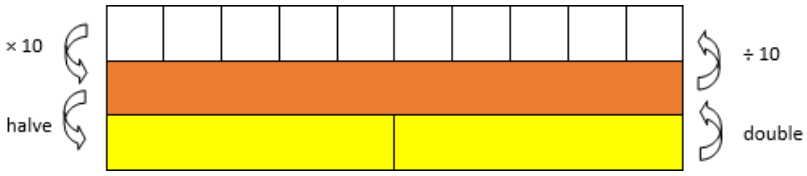
Y5 and Y6 Multiplication

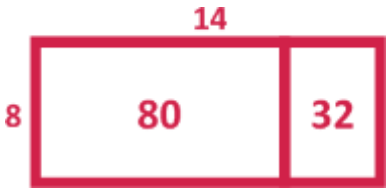
Strategies & Guidance	CPA
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 *Avoid saying that you "add a zero" when multiplying by ten and instead use the language of place holder.* *Use place value counters and charts to visualise and then notice what happens to the digits.*	When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater. 102.14 x 10 = 1021.4  When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller. When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21 210.3 ÷ 10 = 21.03 

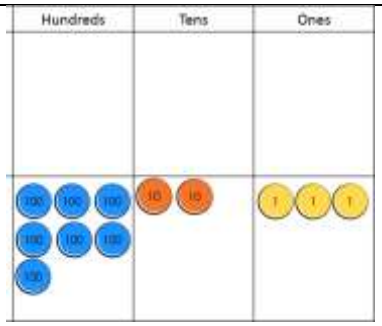
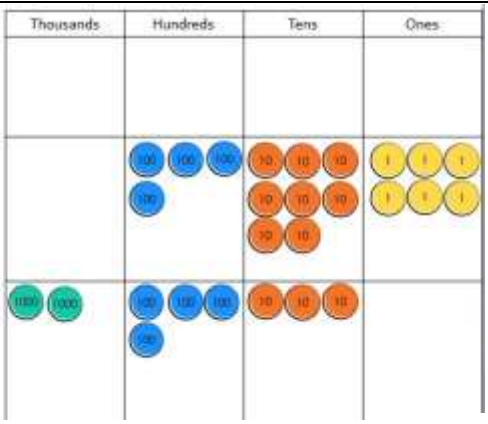
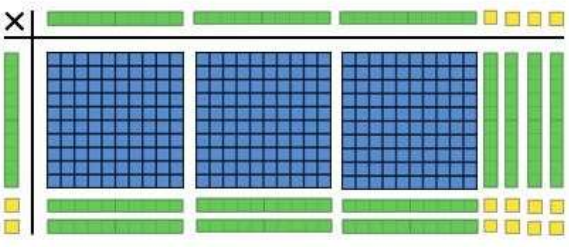
Strategies & Guidance	CPA
Using known facts and place value to derive multiplication facts *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity is further extended and applied to find a range of related facts.* *Pupils should work with decimals with up to two decimal places.* *These derived facts should be used to estimate and check answers to calculations.*	  2 x 3 = 6 3 x 2 = 6 2 x 30 = 60 30 x 2 = 60   2 x 3 x 10= 60 3 x 20 = 60 20 x 3 = 60 2 x 300 = 600 300 x 2 = 600  3 x 200 = 600 200 x 3 = 600 2 x 30 x 10 = 600 20 x 3 x 10 = 600   20 x 30 = 600 30 x 20 = 600

These are the multiplication facts pupils should be able to derive from a known fact

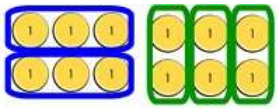
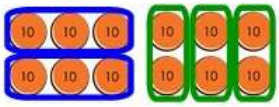
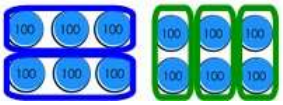
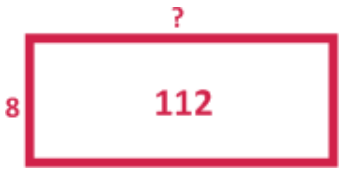
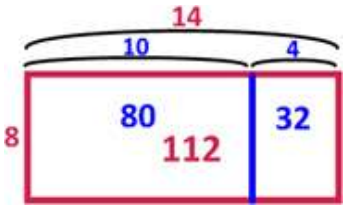
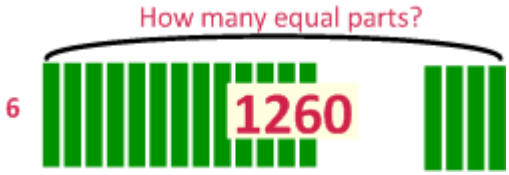
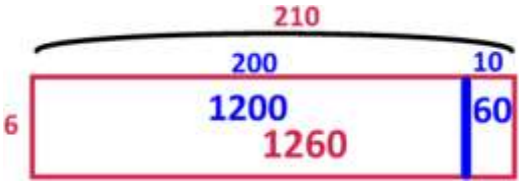
2 100 000		700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000		70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000		7000 x 3	700 x 30	70 x 300	7 x 3000		
2100		700 x 3	70 x 30	7 x 300			
210		70 x 3	7 x 30				
21	**=**	**7 x 3**					
2.1		0.7 x 3	7 x 0.3				
0.21		0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021		0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

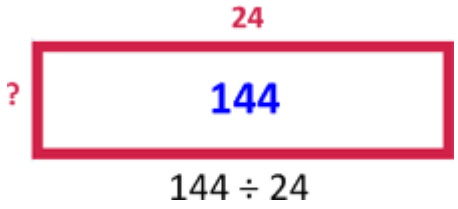
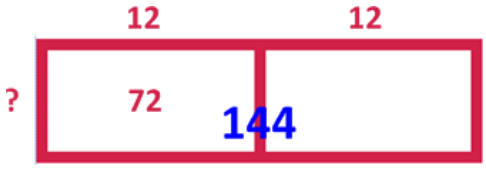
Strategies & Guidance	CPA
Doubling and halving *Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.* *Doubling and halving can then be used in larger calculations.*	 **Multiply by 4** by doubling and doubling again e.g. 16 × 4 = 32 × 2 = 64 **Divide by 4** by halving and halving again e.g. 104 ÷ 4 = 52 ÷ 2 = 26  **Multiply by 8** by doubling three times e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96 **Divide by 8** by halving three times e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13  **Multiply by 5** by multiplying by 10 then halving, e.g. 18 × 5 = 180 ÷ 2 = 90. **Divide by 5** by dividing by 10 and doubling, e.g. 460 ÷ 5 = double 46 = 92

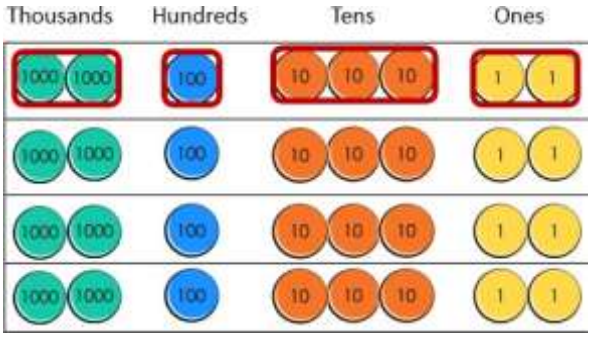
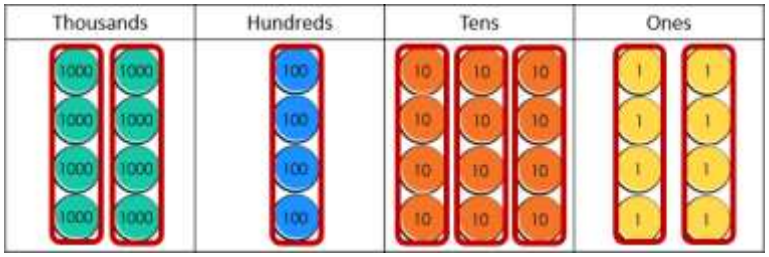
Strategies & Guidance	CPA
Multiply by partitioning one number and multiplying each part **Distributive law** **a x (b + c) = a x b + a x c** Build on pupils' understanding of arrays of counters to represent multiplication to see that area models can be a useful representation:	8 x 14 = 8 x 10 + 8 x 4  Cuisenaire rods to build arrays  Represent with area model  Jottings on a number line Bead string where each bead has a value of 8: 
Using knowledge of factors *In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate.* *Pupils will be using the commutative and associative laws of multiplication.* **Commutative law** **a x b = b x a** **Associative law** **a x b x c = (a x b) x c** **= a x (b x c)** *They should explore and compare the different options and choose the most efficient order to complete calculations.*	Calculate 6 x 24 by using factor pairs of 24 Two and twelve are factors of 24: 6 x 2 x 12 6 x 12 x 2   Three and eight are factors of 24: 6 x 3 x 8 6 x 8 x 3   Four and six are factors of 24: 6 x 4 x 6 6 x 6 x 4  

Strategies & Guidance	CPA
Formal written method of short multiplication *Conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.*	241 x 3 723 1 
Multiplying by a 2-digit number **Formal written method of long multiplication** *In Year 6 pupils are extended from multiplication by a 1-digit number to multiplication by a 2-digit number.* *Extend the place value chart model used in Year 4, using an additional row on the place value chart.* *Extend understanding of the distrubitive law to develop conceptual understanding of the two rows of the formal written method.* *Dienes blocks can be used to construct area models to represent this.*	243 x 12 486 2430  243 x 2 243 x 10 34 × 12 68 340 408 12 × 34  10 × 34 = 340 2 × 34 = 68

Y5 and Y6 Division

Strategies & Guidance	CPA
Deriving facts from known facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	6 ÷ 2 = 3  6 ÷ 3 = 2 60 ÷ 2 = 30 60 ÷ 30 = 2  60 ÷ 3 = 20 60 ÷ 20 = 3 600 ÷ 2 = 300 600 ÷ 300 = 2  600 ÷ 3 = 200 600 ÷ 200 = 3
Using knowledge of multiples to divide *Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).*	 112 ÷ 8 = 80 ÷ 8 + 32 ÷ 8   1260 ÷ 6 = 1200 ÷ 6 + 60 ÷ 6  

Strategies & Guidance	CPA
Using knowledge of factors to divide *Pupils explore this strategy when using repeated halving.* *2 x 2 = 4 and so if you divide by 4 the same result can be achieved by dividing by two and then by two again.*	 144 ÷ 24 I know 2 and 12 are a factor pair of 24 and so I can divide by 2 and then by 12.  144 ÷ 2 ÷ 12

Strategies & Guidance	CPA
Short division **Dividing a 4-digit numbers by 1-digit numbers** *The thought process of the traditional algorithm is as follows:* *How many 4s in 8? 2* *How many 4s in 5? 1 with 1 remaining so regroup.* *How many 4s in 12? 3* *How many 4s in 8? 2* *Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.* *How many 4s in 8000? 2000* *How many 4s in 500?* *100 with 1 remaining (illogical)* *The answer would be 125.* *Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm.* *Using place value counters and finding groups of the divisor for each power of ten will build conceptual understanding of the short division algorithm.* *Area models are also useful representations, as seen with other strategies and exemplified for long division.*	8528 ÷ 4 2 1 3 2 4 | 8 5¹2 8 **Sharing**  8 thousands shared into 4 equal groups 5 hundreds shared into 4 equal groups Regroup 1 hundred for 10 tens 12 tens shared into 4 equal groups 8 ones shared into 4 equal groups. **Grouping**  How many groups of 4 thousands in 8 thousands? How many groups of 4 hundreds in 5 hundreds? Regroup 1 hundred for 10 tens. How many groups of 4 tens in 12 tens? How many groups of 4 ones in 8 ones?

Strategies & Guidance	CPA
Long division **Dividing a 4-digit number by a 2-digit number** *Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.* *If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.*